

Part III

Content Domain, Context, and Culture

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PERSPECTIVES ON EMOTION IN MATHEMATICAL ENGAGEMENT, LEARNING, AND PROBLEM SOLVING

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This chapter addresses research on emotion specific to mathematics or situated in mathematical activity. I hope to highlight three essential ideas: the *centrality* of emotions in mathematical learning and teaching, the *complexity* with which they interact with mathematics-related cognition, and the *domain-specificity* of their occurrence and their influences.

Despite increasing interest over the last two decades, the wider mathematics education research community has but begun to address the affective domain theoretically. Comprehensive volumes (e.g., Kelly & Lesh, 2000; Sriraman & English, 2010; Steffe, Neshet, Cobb, Goldin, & Greer, 1996) devote few pages to affect generally, or to emotion in particular. However, the superb review of teachers' beliefs and affect by Philipp (2007) includes work on math anxiety, attitude-based research, and the study of mathematical emotions and engagement.

McLeod (1989, 1992, 1994), who greatly influenced research on the affective domain in mathematics education, describes it as comprised of *emotions*, *attitudes*, and *beliefs*. In much subsequent work, emotions are studied as they connect with mathematical attitudes or beliefs, including self-theories (Dweck, 2000). DeBellis and Goldin (2006) distinguish *values* (what one holds as important, or cherishes) from beliefs. Pekrun and Linnenbrink-Garcia (2012) note that the term "affect" in emotion research refers more specifically to emotions and moods; here, I use the term in the broad sense prevalent in mathematics education. I seek to focus as specifically as possible on emotions but aim to respect the fact that many in mathematics education regard their importance as being partly or mainly through their connection with attitudes, beliefs, and values.

This chapter considers first the complexity of emotion situated in diverse mathematical contexts and some frequently occurring patterns. Second, some important domain-specific issues pertaining to emotions in mathematics education are discussed, including the role of *impasse*, the frequently occurring disconnection of procedural from

conceptual knowledge and the prevalence in the wider culture of certain beliefs about mathematics and mathematical ability.

The third section contrasts distinct interpretations of emotion in mathematics education: *traits* (characterizing different individuals' most typical emotional responses in mathematical situations), and *states* (emotions as they occur in-the-moment when doing mathematics). These enter into the *architecture* (the nature and functions of emotions in interaction with other affective, cognitive, or social constructs in mathematical environments). Such interpretations sometimes appear as competing or opposed research emphases, but here I regard them as fully compatible if appropriately distinguished.

The fourth section surveys some empirical findings pertaining to *math anxiety*, the most studied mathematical emotion. The larger scale, quantitative research on math anxiety and other emotions in mathematics has tended to focus on its trait-like interpretation. The results illustrate both the value and inherent limitations of studying trait emotions through questionnaire methods. The fifth section discusses research focusing on state-like interpretations, where the predominant mode has been to report qualitative descriptions and illustrative examples—suggesting important theoretical ideas but entailing a different set of inherent limitations.

The last section highlights theoretical ideas about affective architecture, particularly important to mathematics education, which, in my view, should be central to future research.

THE COMPLEXITY OF MATHEMATICALLY SITUATED EMOTION

Mathematics classrooms present an extraordinary variety of contexts for students' emotional experiences. During individual work, small-group problem solving, or whole-class activity, a student may be presenting or listening, following directions, just thinking, or attending to something other than mathematics. Activity may be routine or cognitively challenging, with conceptions correct or incorrect, incomplete, confused, or non-standard. Technology tools require additional, domain-specific skills. The experienced social environment exerts "press" through teacher and peer expectations and immediate events. Different students experience each social interaction differently, as personality traits vary. Events outside school, related or unrelated to mathematics, affect emotional responses. Students' emotions in similar contexts differ sharply. And the contexts for teachers' emotions are just as diverse—different comfort levels with the topics they teach, challenging classroom situations, demands associated, ~~and~~ standardized tests, and so forth. In this manifold of mathematically related contexts, the emotions of different individuals also interact dynamically with each other: some labeled positive (e.g., curiosity, enthusiasm, fascination, love, pleasure, pride, satisfaction); some negative (e.g., anger, anxiety, boredom, fear, frustration, hatred, humiliation); and some variable or harder to classify (e.g., surprise).

Evidence for such emotion comes from questionnaires, from coding and analyzing expressions of emotion in videotaped activity, and other sources. Pekrun's Achievement Emotions Questionnaire–Mathematics (AEQ-M) (Frenzel, Pekrun, & Goetz, 2007; Pekrun, Goetz, Frenzel, Barchfeld, & Perry, 2011) includes items pertaining to enjoyment, pride, anxiety or fear, shame, anger, boredom, and hopelessness. The coding scheme of Else-Quest, Hyde, and Hejmadi (2008) identifies positive interest (described as involving interest, eagerness), tension (involving nervousness, anxiety, worry), frustration,

sadness, anger, boredom, contempt, joy/pleasure, pride, and other emotions and related behaviors. Note that these extend beyond the “basic emotions” (Ekman, 1992; Ekman & Friesen, 2003) of anger, fear, joy, sadness, disgust, and surprise. Op’t Eynde, De Corte, and Verschaffel (2006) consider facial actions, vocalizations, bodily actions, and most importantly, subjects’ retrospective appraisals. They identify emotional sequences—for example, in one episode (Op’t Eynde & Hannula, 2006), worry is followed by frustration, panic, and anger, but ultimately happiness.


We see also in these descriptions variation along other well-known dimensions—from mild (e.g., worry) to intense (e.g., panic) and from activating (e.g., eagerness) to deactivating (e.g., boredom). Some felt emotion can have depth and importance to the individual, while other emotions may be only fleetingly meaningful. And emotions can recur—Else-Quest et al. (2008) report children most often expressing tension and positive interest; sadness, boredom, anger, contempt, affection, joy, humor, and pride occurred on average less than once per session. Of course, such findings are highly dependent on the study’s context (in this case, mother–child out-of-school mathematics sessions with American children, mean age 11.4 years, who had completed fifth grade).

Despite such complexities, certain *patterns* or regularities in emotions and their influence seem to stand out in mathematics education. Some not only jibe with plausible expectation but are substantiated by quantitative research—for example, the direct relation of math anxiety to students’ perceptions of their math ability and their objective performance (e.g., Meece, Wigfield, & Eccles, 1990; see below). “Attitude toward mathematics,” taken to have affective, behavioral, and motivational components, may include a propensity toward emotions such as enjoyment, liking, or the absence of boredom, as well as toward approach (vs. avoidance) behaviors. Favorable attitude (implicitly, positive emotion) is associated with school achievement (e.g., Mullis et al., 2008). Such identifiable, persistent, and widespread correlations of emotions with important goals of mathematics education constitute part of “what we know.” But the details of the interactions among emotions, behaviors, and motivational orientations are crucial and should not be glossed over just by defining attitude as a composite.

Emotional sequence patterns are reported in qualitative studies in particular contexts. More complex patterns—for example, idealized *affective pathways* where sequences of emotional states interact with heuristics during mathematical problem solving (Goldin, 2000)—are proposed as plausible theoretical conjectures based on qualitative observations and teachers’ and students’ widely shared experiences. Some patterns—such as the association of mathematics with painful experience—also manifest themselves in the media and the wider culture.

There is much to learn about how in-the-moment emotions that students experience during mathematical activity contribute to longer term effects and about how teachers may skillfully influence them. For example, a student may become angry when another student says that her group’s method of solving a problem is wrong (e.g., Schorr, Epstein, Warner, & Arias, 2010a, 2010b)—but public challenges to a student’s ideas by the teacher or a peer can evoke on one occasion defensiveness, anger, or humiliation, and on another, excitement or determination. Such examples of situated emotion and accompanying behavior patterns are familiar to mathematics teachers, and their characterization is essential to understanding how emotions affect students’ longer term mathematical development. But we must take into account how at different times a person experiences different emotional sequences, even in similar circumstances.

DOMAIN-SPECIFIC FEATURES

Let us next consider certain features with emotional implications that may be particular to mathematics as compared with other school subjects. I would highlight: (a) the central role of *impasse*, (b) the frequently occurring disconnection of procedural from conceptual knowledge, ,

(c) embedded conceptual challenges, (d) the hierarchical nature of the curriculum, (e) the importance of correct answers and the frequent unreliability of attaining them, and (f) the prevalence in the wider culture of certain beliefs about mathematics and mathematical ability.

Problem solving is central to mathematical activity. But a problem is a situation in which the person has a goal but does not immediately know how to reach it (Schoenfeld, 1985), experiencing some kind of impasse or obstacle, some cognitive incongruity not easily resolved. Such impasse is likely to evoke *bewilderment*, or if it persists, *frustration* and an accompanying spectrum of emotions, from anxiety to increased interest and curiosity (Goldin, 2000; see also D’Mello & Graesser, 2014). Op ’t Eynde, De Corte, and Verschaffel (2007) report students feeling annoyance, anger, anxiety, frustration, nervousness, happiness, and relief during problem solving, with frustration and nervousness occurring most frequently.

Mathematics involves procedures (rules for symbol-manipulation) as well as concepts (meanings, representations, and interpretations including why procedures work) (Lesh & Landau, 1983). Skemp (1976) calls these “instrumental” and “relational” understanding respectively. Tests often focus on fluency in arithmetic algorithms or algebraic manipulations, procedures that can often be acquired with minimal conceptual understanding (cf. Lesh & Lamon, 1992). But teaching well-established routines to ensure skills proficiency or to increase test performance (e.g., Firestone, Schorr, & Monfils, 2004; Handal, 2003; Ma, 1999; Smith, 1996) may leave some students *bored* and *disinterested* (Mora, 2011). Performance disconnected from concepts can lead to *discomfort*, *dislike*, and/or *anxiety* as the student follows rules without knowing why she is to do so (e.g., Nardi & Steward, 2003) or to *satisfaction* when and if she acquires a relational understanding (or succeeds using an algorithm with only instrumental understanding). Concept development in mathematics requires pressure-free exploration and discussion time, the unavailability of which in school can evoke *frustration*, while achieving conceptual understanding may lead to *elation*.

When new concepts (e.g., fractions, negative numbers, unknowns in algebra, formal geometry proofs, functions, limits, derivatives, and integrals) are first introduced, they typically require cognitive restructuring, the reinterpretation of existing representations or construction of new ones (e.g., Davis, 1984). This may lead to *confusion* and self-doubt or to *pride*, *satisfaction*, *appreciation*, and *self-confidence*, according to the degree of success and the social environment (e.g., Lewis, 2011, 2012; McCulloch, 2011; Schorr & Goldin, 2008).

The mathematics curriculum and its subfields (algebra, geometry, analysis, etc.) are typically organized hierarchically, so that failure to master prior concepts and prerequisite skills impedes subsequent learning. *Discouragement* may occur and a sense of falling behind when, for whatever reason, the learning sequence is interrupted.

Mathematics involves frequent evaluation of students’ work as correct or incorrect, providing negative as well as positive feedback. Such evaluation may lead alternately

to *elation* and *disappointment*. Forsyth (1986) describes a range of emotional reactions to examination scores, with failing students experiencing *unhappiness*, *tension*, and *guilt*. If the context is competitive or public, emotions easily extend to *pride* or *humiliation*. But mathematical correctness has an unreliable aspect even when the concepts and procedures are well understood—namely, the likelihood of oversight, clerical error, or miscommunication. Mathematical notation is highly nonredundant, so that a single misplaced character changes an expression's meaning. Many routine mathematical steps are expected to be taken mentally. Thus, the student has limited control of the outcome, leading possibly to *frustration* and a sense of *despair*.

Finally, certain prevailing, mathematics-specific beliefs can meet emotional needs, providing *comfort*, a sense of *security*, and some justification for experienced emotion and/or defenses from pain (Handal, 2003; Leder, Pehkonen, & Törner, 2002; Maasz & Schlöglmann, 2009). Mathematics is widely seen as requiring special ability, intelligence, or genius, often believed to be inherited or innate. Such beliefs, reflected in educational practice in many countries, may affect self-expectations and the expectations of others, influencing in turn success or failure emotions—for example, helping protect an unsuccessful student (as well as his teacher) from feeling *guilt*, *frustration*, or *despondency*, as lack of success is then neither one's fault (Goldin, Rösken, & Törner, 2009, p. 11). Alternatively, it may offer someone a sense of *pride* and family connection in being “mathematically gifted.”

Mathematics is often believed to be purely rational, so that emotion is irrelevant—encouraging its suppression. Historically, mathematics has been male-dominated, with a continuing undercurrent of belief that women are less able than men to excel in it. Black and Hispanic students in the United States are greatly underrepresented in mathematical fields. These aspects may lead to phenomena such as stereotype threat (Aronson et al., 1999; Steele & Aronson, 1995), where consequent emotions inhibit performance.

TRAITS, STATES, AND ARCHITECTURE

Next, let us distinguish explicitly two different interpretations of what one means by emotions, and the different sorts of research questions and methods those interpretations suggest.

Trait-Like and State-Like Interpretations

The distinction between *state* (a person's in-the-moment psychological particulars, which can change rapidly) and *trait* (a longer term, relatively stable characteristic) is long-standing in the psychology of personality (Cattell & Scheier, 1961). Emotional states involve highly variable, situation- and event-dependent feelings (see Shuman & Scherer, 2014; Turner & Trucano, 2014). *Mood* states change less rapidly and may also be less specifically attached to an identifiable cause or referent (cf. Linnenbrink & Pintrich, 2004). Emotional traits refer to how someone typically feels and how his or her feelings characteristically differ from someone else's (e.g., Izard, 1991). The term *local affect* (Goldin, 2000; Gómez-Chacón, 2000) includes state emotions and mood states but also their moment-by-moment interactions with cognition, with the social environment, with the emotions of others, and with the individual's traits. *Global affect* includes trait emotions as well as stable structures that incorporate emotions—not only attitudes, beliefs, and values, but constructs such as mathematical self-identity.

The preponderance of large-scale questionnaire-based research in mathematics education has focused on trait emotions. Some instruments treat such emotions as components of attitudes or orientations (e.g., Fennema & Sherman, 1976); others, like the Math Anxiety Questionnaire (MAQ), address emotions directly (Wigfield & Meece, 1988). Trait-like emotions are not necessarily defined to be as enduring as the term trait might suggest. Thus, when Frenzel et al. (2007) use the AEQ-M to study German high school students' enjoyment, anxiety, anger, and boredom, the contextualized questions might be read to refer to emotions typically felt just that year or that term in connection with school mathematics, and not necessarily longer lasting. The MAQ item, "I dread having to do math," suggests a more permanent domain-specific trait emotion than the AEQ-M item, "I enjoy my math class." Trait emotions can also be assessed through interviews and field observations (cf. Tobias, 1993), although large-scale qualitative studies of trait emotions are more costly and therefore rare. Research goals include measuring correlations (positive or negative) between trait emotions and mathematical engagement, learning, and problem solving success, studying their association with age, gender, or other population characteristics, characterizing the underlying structure of the traits themselves, identifying their origins, and discovering how one may influence them through interventions.

Techniques for the study of state emotions in the mathematics education literature most often feature inference and analysis from close, in-the-moment observation (usually, but not always, qualitative analysis)—for example, videotaped classes or task-based and retrospective interviews, including stimulated recall interviews (e.g., Zan, Brown, Evans, & Hannula, 2006). The researcher seeks to infer, often with considerable uncertainty or unreliability, the shifting emotions such as curiosity, frustration, anger, anxiety, elation, or satisfaction actually felt in particular situations—their origins, functions, and consequences. Experience sampling methods (ESM) have been used less frequently (e.g., Schiefele & Csikszentmihalyi, 1995), but ESM is likely to become more influential as clickers and more sophisticated mobile devices come into use. Employing questionnaire methods to study state emotions in authentic mathematical contexts is more difficult; to ask state-anxiety questions such as "find the word or phrase that best describes how you feel right now, *at this very moment*" (Spielberger, Edwards, Montuori, & Lushene, 1973, emphasis in original) is impractical during engaged activity without disruption. Questionnaires given immediately after activity can provide insight into emotional states, but few such studies have thus far been done in mathematics education. Research goals include understanding and modeling how and why emotions in students or teachers arise, their relation to problem solving, learning, and teaching, and how they influence or are influenced by mathematical motivation, achievement, attitudes, beliefs, or other variables. Recurrent patterns in state emotions invite theoretical characterization, with potential for effective teacher interventions—strategies for turning in-the-moment emotions toward constructive learning goals, even without detailed knowledge of individual students' traits. Parallel comments can be made about mathematics teachers' emotions.

Sometimes these two strands seem to exemplify conflicting research paradigms (see discussion further on). But the research questions asked about emotion are quite parallel, albeit in different time frames and on different levels of situation-specificity. A contention in this chapter is that they can contribute in a mutually consilient way to a unifying theory of mathematical affect.

An interesting question is how to interpret the long-term *recollection* of earlier emotion in mathematics (e.g., Karsenty, 2004). The emotions recalled are, in principle, prior

states; the feelings reported during recall are current state emotions. Yet, the possibly selective recall of emotion after a long interval and the incorporation of emotions accompanying such recall into what Karsenty terms “mathematical self-schema,” suggests that they also have trait-like aspects.

Affective Architecture

Both state and trait emotions form a part of the architecture of affect. Architecture refers to the universal or near-universal functions of emotion, including structures within which emotions occur in human beings: how emotions are constituted, how they link with cognition, attitudes, beliefs, or values, social interactions, cultural norms and roles, and engagement (cf. Pekrun & Linnenbrink-Garcia, 2012), how they encode information, their communicative function and its relevance to cooperation or competition, and their domain-specificity versus generality (e.g., Goetz, Frenzel, Pekrun, Hall, & Lüdtke, 2007), reciprocal aspects of emotions, the role of meta-affect (see further on), and so forth. The focus here is neither on identifying traits nor describing states but on the nature and mechanisms of the interactions between emotions and mathematical learning, teaching, and problem solving.

Note that the words we use for emotions have different meanings when interpreted as states or traits, or when taken to be descriptive of architecture. “The student is *angry* because he missed solving an algebra problem,” describes (partially) his state. “The student is *in an angry mood*,” whether triggered by her mathematics test result or for an unidentifiable reason, suggests a state likely to persist. “He has a lot of *anger toward mathematics*,” describes (partially) an emotional *trait*; he may not feel angry now, but he will probably continue to have a lot of anger toward mathematics. “He is an *angry person*,” suggests a much less domain-specific emotional trait. But to say, “*Anger* [as a human emotion] combines strong disapproval and distress” (paraphrasing Ortony, Clore, & Collins, 1988, pp. 146–149) is to make a theoretical assertion about the nature of anger in general, and its possible place (as a compound, in this description) in the spectrum of human emotion. No person is mentioned, nor is mathematics mentioned, but ways of interpreting a person’s anger in mathematical contexts are strongly implied.

Questions posed in mathematics education—for example, how curiosity, bewilderment, frustration, or relief interact with strategic decisions during problem solving (e.g., DeBellis & Goldin, 2006), addressed through qualitative analyses of individual episodes, how achievement emotions such as enjoyment anxiety, anger, or boredom relate to perceptions of mathematics classroom contexts (e.g., Frenzel et al., 2007; Pekrun, 2006), addressed through large-scale studies, or how anxiety, comfort, or satisfaction influence and help sustain a student’s or teacher’s beliefs about her mathematical ability (e.g., Leder et al., 2002; Philipp, 2007), addressed through surveys or interviews—rest on assumptions or theories about the architecture of affect. Their answers, of course, require empirical research on state and/or trait emotions.

Competing Emphases in Mathematics Education

One of the dichotomies in mathematics education research has been a tension between the focus on obtaining broad, generalizable findings about the occurrence of various emotions and their correlates, with an eye to developing structural models descriptive of

populations (but possibly disregarding essential complexities), and the focus on understanding and influencing the complex, dynamic emotional episodes that occur during learning and problem solving (but possibly disregarding the goal of scientific generalizability). Prior to the late 1980s and early 1990s, most research on mathematical affect centered on the development and measurement of attitudes (e.g., Fennema & Sherman, 1976) and/or math anxiety (e.g., Richardson & Suinn, 1972) and their relation to learning and performance—that is, trait-like emotion. This work goes on, and some findings are summarized below. However, a growing segment of the research community questions the value of such research. For example, Zan et al. (2006, p. 114) remark that the theory underlying it is not only limited but drawn mainly from other disciplines:

The driving force in much research seemed to be “the statistical methodology rather than the theory” (McLeod, 1987); researchers rarely gave explicit definitions of their construct, often leaving the definition to be inferred from the type of instrument used. This lack of conceptual clarity was related to the borrowing of instruments and constructs from psychology, without specific theoretical elaboration for mathematics education.

McLeod’s evident intent, as well as that of Zan et al., is to criticize the questionnaire-based study of attitude and of trait emotions of the kind summarized below for math anxiety. Inspired partly by successes in the cognitive analysis of problem solving (e.g., Schoenfeld, 1985), McLeod had proposed to focus on the qualitative, fine-grained study of affect. His characterization took emotions, attitudes, and beliefs to be in order of increasing temporal stability and linkage with cognition, and decreasing intensity, thus positing a certain architecture of affect—emotions exclusively as states (with trait emotions incorporated into attitudes). This encouraged a then-new direction in mathematics education—studying in-the-moment emotion in its own right, distinct from longer term constructs. During the subsequent decades, with few exceptions, there seems to have been quite little interaction between those continuing to study trait emotion, using questionnaires and to a much lesser extent qualitative interviews (e.g., Ho et al., 2000; Jain & Dowson, 2009), and those focusing on the fine-grained analysis of state emotion, using qualitative methods (e.g., Gómez-Chacón, 2000; Hannula, 2002, 2006).

To exemplify the study of a trait emotion, let us focus next on the most well-studied domain of emotion in mathematics education—that of students’ math anxiety.

THE STUDY OF A TRAIT-LIKE EMOTION TOWARD MATHEMATICS: ANXIETY

Anxiety is a widespread, negative emotion promoting aversion to mathematics (e.g., Baloğlu & Koçak, 2006; Tobias, 1993) in which gender differences are also found (Devine, Fawcett, Szűcs, & Dowker, 2012). The apparent prevalence of anxiety in relation to mathematics (and/or manifestations such as unease, nervousness, and apprehension, or related emotions such as fear or unhappiness) favors its research study as a trait emotion.

Beasley, Long, and Natali (2001) enumerate various measures of mathematics anxiety. The Fennema-Sherman Mathematics Attitude Scale (Fennema & Sherman, 1976) incorporates nine different Likert-type attitude scales hypothesized as important either for all students or for females specifically; the “Mathematics Anxiety Scale” is one. The

Mathematics Anxiety Rating Scale (MARS) takes different forms (Plake & Parker, 1982; Richardson & Suinn, 1972; Suinn, 1988); the original consists of 98 items designed to assess anxiety in situations involving numbers and mathematical problems using a five-point Likert-type scale. The Mathematics Anxiety Scale for Children (MASC) is a shortened version, correlating highly with the MARS, designed for younger children with shorter attention spans (Chiu & Henry, 1990). The Math Anxiety Questionnaire (MAQ) items reported by Wigfield & Meece (1988) include 11 items addressing worry, uneasiness or nervousness, and fear, scored on a seven-point scale. They omit four items that prior factor analysis identified as assessing dislike of mathematics as distinct from anxiety.

A meta-analysis by Hembree (1990) of 151 studies of mathematics anxiety using validated instruments identifies correlating variables, reports population variables exhibiting different levels of mathematics anxiety, considers the relation between mathematics anxiety and performance, and examines the effects of treatments. Higher math anxiety correlates inversely with mathematical performance, variously measured, at all grade levels, with mean correlations of $r = -.36$ ($p < .01$) for males and $r = -.30$ ($p < .01$) for females in grades 5–12. It correlates inversely with the intentions by students in grades 7–12 to take more math ($r = -.35$, $p < .01$ for males, $r = -.25$, $p < .01$ for females). Much greater negative correlations are found between math anxiety and other attitude-related variables, such as enjoyment of math in grades 5–12 (mean $r = -.75$, $p < .01$), self-confidence in math in grades 6–11 ($r = -.82$, $p < .01$, a high negative value derived from 4 studies involving 514 subjects), self-concept in math ($r = -.71$, $p < .01$), and motivation in math ($r = -.64$, $p < .01$).

Correlations of math anxiety with other measures of anxiety are positive at the post-secondary level (where most such studies were conducted): with trait anxiety ($r = .38$, $p < .01$), state anxiety ($r = .42$, $p < .01$), and test anxiety ($r = .52$, $p < .01$). Math anxiety is higher for females than males at all grade levels according to Hembree's analysis, generally increasing from middle school into grades 9–10 and then leveling off. Some interventions (e.g., systematic desensitization, cognitive-behavioral) are highly successful in achieving math anxiety reduction, with significant positive effects on mathematics test performance.

Hembree (1990) believes there is evidence that math anxiety reduces performance, but “no compelling evidence that poor performance causes math anxiety” (p. 44). Comparing math anxiety with (general) test anxiety, he concludes, “only 37 percent of one construct's variance is predictable from the variance of the other. . . . Hence, it seems unlikely that mathematics anxiety is purely restricted to testing. Rather the construct appears to comprise a general fear of contact with mathematics, including classes, homework, and tests” (p. 45).

A subsequent meta-analysis by Ma (1999) addresses the relationship of mathematics anxiety and mathematical achievement in 26 studies of students in grades 5–12, both published and unpublished. Ma (1999) determines that “published studies tended to indicate a significantly weaker relationship than unpublished articles” (p. 531), while:

the common population correlation for the relationship between anxiety toward mathematics and achievement in mathematics was $-.27$ Results show that [this relationship] is consistent across gender groups (male, female, and mixed), grade-level groups (Grades 4 through 6, Grades 7 through 9, and Grades 10 through 12), ethnic groups (mixed and unmixed), instruments used to measure anxiety (MARS and others), and years of publication. . . . Researchers using standardized achievement

tests tended to report a significantly weaker relationship than those using researcher-made achievement tests and mathematics teachers' grades.

(Ma, 1999, p. 531)

Lee (2009) reports the correlations with mathematics scores and conducts factor analyses of the self-constructs of math self-concept, math self-efficacy, and math anxiety based on the 2003 Program for International Student Assessment (PISA) data. Participants included over 250,000 15 year olds from 41 participating countries. Within-country correlations of math anxiety with math scores range from $r = -.51$ (Denmark) to $r = -.12$ (Indonesia), mean correlation $r = -.39$; for comparison, the mean correlation for math self-efficacy is $r = .43$ and for math self-concept $r = .23$; all correlations are significant ($p < .01$). The between-country correlation for math anxiety with math score was large ($r = -.65, p < .001$), greater than that for math self-efficacy ($r = .42, p < .001$), while the between country correlation for math self-concept was negative ($r = -.45, p < .001$). In addition, factor-analytic results seem to well support the hypothesis that math self-concept, math self-efficacy, and math anxiety are separate, empirically distinguishable constructs across and within countries.

Math anxiety as measured may or may not consist of more than one factor. In a cross-national study of 671 sixth-grade students in China, Taiwan, and the United States, the distinction between affective and cognitive dimensions of math anxiety was supported in each of the three populations, with the affective factor negatively related to achievement (Ho et al., 2000). Here the analogy is with test anxiety (see Zeidner, 2014); the affective factor refers to “the emotional component of anxiety, feelings of nervousness, tension, dread, fear, and unpleasant physiological reactions” while the cognitive factor is defined as “the worry component of anxiety, which is often displayed through negative expectations, preoccupation with and self-deprecatory thoughts about an anxiety-causing situation” (Ho et al., 2000, p. 363). Other authors have suggested as many as six factors; for example, Bessant (1995) gives an 80-item version of the MARS to 173 college students and identifies dimensions labeled General Evaluation Anxiety, Everyday Numerical Anxiety, Passive Observation Anxiety, Performance Anxiety, Mathematics Test Anxiety, and Problem-Solving Anxiety. In contrast, Beasley et al. (2001) conclude from their study of 278 sixth-grade children using the MASC that math anxiety may be unidimensional. Devine et al. (2012) use the Abbreviated Math Anxiety Scale (AMAS) (Hopko, Mahadevan, Bare, & Hunt, 2003) to look for gender differences in the relation of math anxiety to mathematical performance in 433 British students in school years seven, eight, and one. Controlling for test anxiety, they find that math anxiety correlates negatively with performance only for girls.

Rayner, Pitsolantis, and Osana (2009) investigate math anxiety in preservice teachers. Administering the Revised Mathematics Anxiety Rating Scale (RMARS, Baloğlu, 2002) to 32 preservice teachers, as well as instruments designed to assess procedural and conceptual knowledge of fractions, they report that increasing math anxiety was associated with decreasing procedural as well as conceptual knowledge. This finding is against a background of earlier research:

where prospective and practicing teachers were requested to identify the source of their mathematics anxiety, none of the participants attributed his mathematics anxiety to difficulties in recalling mathematical procedures during anxiety-evoking

situations . . . they reported that instruction that was procedurally focused while at the same time lacking in conceptual support was a salient factor that played a role in the development of their mathematics anxiety (Bowd & Brady, 2003; Brady & Bowd, 2005; Harper & Daane, 1998; Trujillo & Hadfield, 1999; Uusimaki & Nason, 2004; Widmer & Chavez, 1982).

(Rayner, Pitsolantis, & Osana, 2009, p. 63)

Moving beyond the study of correlates of mathematics anxiety, researchers have sought to establish structural or causal models (e.g., Akin & Kurbanoglu, 2011; Ma & Xu, 2004; Meece et al., 1990). Sherman and Wither (2003) report on a longitudinal study by Wither, testing whether math anxiety causes impairment of math achievement (as conjectured by Hembree), or absence of math achievement causes math anxiety, or a third condition is responsible for both. The study applies cross-lagged panel analysis to nine pairs of tests of math achievement and math anxiety, administered to a common group of students from three schools in suburban Adelaide, Australia over five years; 96 of the original 156 students completed all nine test sessions. The hypothesis that math anxiety causes a reduction in math achievement is rejected, while the evidence is insufficient to distinguish between the other two possibilities. Jain & Dowson (2009) consider self-regulation and self-efficacy variables in relation to math anxiety, testing a structural equation model based on questionnaire data from 232 eighth-grade students in India. They conclude:

(a) the survey scales represent substantially good measures of the factors they are intended to measure; (b) gender and age can be accurately modeled as influences on self-regulation, self-efficacy, and mathematics anxiety; and (c) mathematics anxiety can be accurately modeled as an outcome of multidimensional self-regulation mediated by self-efficacy.

(Jain & Dowson, 2009, p. 245)

The last conclusion differs from, but does not necessarily contradict, Lee's (2009) result that self-efficacy and math anxiety are consistently empirically distinguishable. A valuable recent bibliographic source on math anxiety is Devine et al. (2012).

Studies of positive trait emotions toward mathematics, such as enjoyment, are less extensive. The 2003 PISA data include a scale measuring interest and enjoyment; on average, this trait accounts for only 1.5% of the variance in students' mathematics performance (with greater enjoyment associated with better performance scores), although within certain countries the correlation is greater—as much as 15.5% of the variance in Korea and 16.1% in Norway. In Mexico, Indonesia, and Brazil, the reported relationship (albeit small) is in the opposite direction (OECD, 2004). Overall, only about a third of the study participants reported enjoyment of mathematics, while about half reported interest.

LIMITATIONS

The accessibility of generalizable results about trait emotions—even when correlations are relatively weak—creates a powerful pull toward theories in which trait emotions play the leading roles. The focus then becomes mainly or exclusively students'

characteristic responses toward mathematics and how to develop them positively or change them when they are aversive. The example of math anxiety also suggests how a variable defined by a single trait-like emotion comes to subsume related emotional feelings in its definition—nervousness, frustration, worry, and fear tend to become part of the same construct as it is operationalized through different instruments. Certain features of emotional architecture are suppressed by the assumption (and correspondingly, the abundance of supporting data) that emotions can be interpreted independently of many of the specifics of their context, that possibly distinct negative emotions serve equivalent functions, and that the role of a negative trait emotion such as anxiety is mainly to impede mathematical learning or performance (e.g., Maloney & Beilock, 2012). The difficulty in ascertaining whether math anxiety is comprised of one or several factors exemplifies a limitation in trying to understand the psychological makeup of a trait emotion (in individuals) through patterns of questionnaire responses across populations.

When self-reported emotions are surveyed, positive emotions typically correlate with each other, as do negative emotions (e.g., Laurent et al., 1999). It is then overly easy to reify the positive valence of emotion as the construct most worth studying or to use self-reported positive emotions as one's measure of affective engagement in mathematics and negative emotions as one's measure of disaffection (e.g., Skinner, Kindermann, & Furrer, 2009, adapted from Wellborn, 1991). But the amount of variance in important outcome variables for mathematics education that is attributable to trait emotions remains, at best, modest.

THE STUDY OF IN-THE-MOMENT EMOTION TOWARD MATHEMATICS


Following McLeod's call for fine-grained analyses of affect, there ensued much greater attention to the role of emotional complexity in mathematical problem solving, self-regulation, and motivation (e.g., DeBellis & Goldin, 2006; Goldin, 2000; Gómez-Chacón, 2000; Hannula, 2002; Malmivuori, 2006; Op't Eynde et al., 2006, 2007) and the interaction of emotions with beliefs (e.g., Leder et al., 2002; Maasz & Schlöglmann, 2009; Philipp, 2007). Because studies of this nature describe individual episodes of emotion, the findings usually take the form of detailed descriptions, together with suggested theoretical constructs or conjectures about affective architecture important to mathematics education. Most of this work deemphasizes quantitative methods and is based on videotaped observation and open-ended task-based and retrospective interviews. But in describing this trend, let me make my own opinion clear that both qualitative and quantitative methods are appropriate—and, ultimately, necessary—to the domain-specific study of both emotional traits and states. The method should depend on the questions asked, which in turn depend on the theory underlying the research.

Let us consider just a few examples (out of dozens of relevant investigations). Nardi and Steward (2003) describe a one-year study in England of three Year Nine middle-ability classes (in schools labeled N, C, and T). They conduct classroom observations and group student interviews, coding student statements and considering the frequencies with which certain statements occur. They focus on students whose mathematical engagement appears due mainly to obligation or pressure, with little joy, and seek the "sources of this disaffection" (p. 349). Student responses include many descriptions of

recent or frequently felt emotions. An example illustrates absence of satisfaction in the disconnection of conceptual from procedural learning in mathematics:

Rosanna (N): . . . Yeah, I don't understand it all, like exactly how it would all like work together. I just . . . I'm just like told that that's how you do it but I don't understand how really you do it. I just do it like that.

Interviewer: Right, so it's not . . . so it's not satisfying for you? [Rosanna says yes] Because of that.

Rosanna (N): So you . . . you . . . you know how to do it because you  been told to do it like that but you don't really understand why it's done like that. (Nardi & Steward, 2003, p. 357)

Other statements pertain either to trait emotions or to frequently or recently experienced states, for example: “*I hate maths because I'm not very good at it.* Rebecca (N)” (Nardi & Steward, 2003, p. 357), or “*I want to enjoy maths but I can't because it's so boring.* Noel (T)” (p. 351) (emphases in original). Supported by such examples, Nardi and Steward (2003) profile “quiet disaffection” in mathematics as a composite of “Tedium, Isolation, Rote learning (rule and cue following), Elitism and Depersonalisation” (p. 350), to form the acronym TIREd.

Lewis (2012) reports in detail on interview data with “Helen,” a college student highly disaffected with mathematics. “Her relationship with maths ebbs and flows in tandem with her confidence . . . there is no sense of agency or internal regulation for her confidence or competence ” (Lewis, 2012, p. 117). She describes her feelings as involving (variously) hatred, anger, frustration, humiliation, and boredom associated with low self-efficacy beliefs. Occasional positive emotions are associated with group activity and helping others with the mathematics. Lewis (2012) interprets the case as illustrating “the motivational and emotional complexity of students’ relationship to mathematics” (p. 121); he describes aspects of this complexity using “reversal theory” (Apter, 2001), which involves shifts between oppositional pairs of motivational states.

Efforts to connect longer-term traits with emotional responses during problem solving are exemplified by Op't Eynde et al. (2007), who classify students into “types” according to the positivity/negativity of their belief profiles (based on the authors’ Mathematics-Related Beliefs Questionnaire). They conclude that task-specific perceptions and emotions (including task attractiveness and anxiety) are closely related to students’ beliefs: those with more negative belief profiles generally found tasks less attractive and experienced higher anxiety. They also emphasize the complexity and the context-dependence of affect. They embed self-regulation of emotion in a sociocultural model for mathematical problem solving, where what they term “meta-emotional competencies” are essentially *situated* in classroom contexts: “Students’ competence to self-regulate [their] unpleasant emotions in effective ways might be an important determinant of successful mathematical problem solving” (Op't Eynde et al., 2007, p. 199). Malmivuori (2006) also highlights self-regulatory functions of mathematical affect in relation to the social environment.

DeBellis and Goldin (1993, 1997, 1999, 2006) focus on the interaction of emotion with cognition in videotaped mathematical task-based interviews with children in grades four to six, inferring emotion from children’s statements, interjections, tone of voice, and also using Izard’s (1983) Maximally Discriminative Facial Movement Coding Scheme (MAX). Affective pathways (sequences of emotions interacting with mathemati-

cal cognition) are reported; for example, a boy's surprise/enjoyment blend moving to a possible anger/enjoyment blend as he achieves a mathematical insight inconsistent with his prior expectation (1993, pp. 60–61). McCulloch (2011) studies six high school calculus students using graphing calculators. The calculators help maintain productive affective pathways (e.g., frustration shifting to curiosity/comfort and then to contentment) versus unproductive ones (e.g., comfort shifting to curiosity and nervousness, then to discouragement, then helplessness and annoyance, discomfort, and finally embarrassment) only when their use is *instrumentalized* in the sense of Artigue (2002)—that is, they are progressively transformed from artifacts to instruments through a process of “loading” with potentialities.

Walen and Williams (2002) describe in detail situated emotions of two adult women and one grade three child in the context of timed mathematics tests, drawing some implications regarding inequitable access to mathematics due to timed performance. Their subjects display neither math anxiety nor test anxiety as trait emotions, but the time limit situation evokes great fear. The value placed on speed in the social context of school leads the child to an experience of humiliation.

State emotion emerges as very important to other constructs in mathematics education. Heyd-Metzuyanin and Sfard (2012) study a small group in a grade seven class working on an unfamiliar problem involving fractions. They code the participants' utterances, mapping “the moment-by-moment alterations of the emotional hue . . . aiming at capturing a ‘flow of emotional expressions’ ” (2012, p. 133). After the analysis, they conclude, “Above all, we were struck by the amount and emotional intensity of the subjectifying activity that took place in the classroom. As a result, our whole interpretation of what happened changed” (2012, p. 141). They interpret the episode as exemplifying “identity struggles.”

Our group at Rutgers studies the mathematical engagement of middle school students during small-group, in-class activity, using pre- and post-interviews with teachers, videotaped class activity and small-group activity, retrospective stimulated-recall interviews with selected students, and the use of questionnaires asking about students' desires, thoughts, actions, and emotional feelings during the just-completed math class (Alston et al., 2007; Epstein et al., 2007; Goldin, Epstein, & Schorr, 2007; Goldin et al., 2011; Schorr et al., 2010a, 2010b). Initially, we sought to create a coherent narrative via analysis through four lenses: the flow of mathematical ideas, key affective events (where strong emotion or change in emotion is expressed or inferred), social interactions among the students, and significant teacher interventions; but despite the use of detailed codings, we came to see these perspectives alone as insufficient to understand what was governing students' engagement or disengagement. In one episode (Epstein et al., 2007), a short boy [“Will”] crumpled and threw away the paper on which he had written his solution to the mathematics problem under discussion, and he shared his ideas only reluctantly:

[Will:] . . . mine could have been wrong, and theirs could have just been right. So, if they had chosen my wrong one, and the right one they tossed it away, they might'd get mad at me. So I just left it like that . . .

[Describing] how he was feeling when he crumpled the paper, he said that his “level of happiness went down.”

[Will]: I didn't like, saying anything. [Int]: Why not? [Will]: Because, it might just cause an argument in the first place. [Int]: And how do you feel when there's an

argument? [Will]: I don't like arguing with people, because mostly, they become more like a fight.

(Epstein et al., 2007, p. 654)

Based on the qualitative analysis of numerous videotaped episodes, we identify several recurring patterns—"behavioral/affective/social constellations"—of in-the-moment desires, emotions, behaviors, and social interactions, which we term *engagement structures* (Goldin et al., 2011).

In analogy with cognitive structures, engagement structures are situated in the individual and become active in certain social/mathematical situations. Each is comprised of as many as 10 interwoven, mutually interacting strands that characterize it: (a) an immediate goal or motivating desire, (b) a pattern or patterns of behavior toward fulfilling the desire, including social interactions, (c) a sequence of emotional states (affective pathway), (d) expressions of affect by the person, (e) meanings that the emotional feelings encode, (f) meta-affect, (g) self-talk or inner speech, (h) interactions with systems of beliefs and values, (i) interactions with attitudes and other longer term traits, and (j) interactions with problem-solving strategies and heuristics.

Examples of engagement structures (and the corresponding motivating desires, which lend their names to the structures) include: (a) Get The Job Done: the desire is to complete an assigned mathematical task correctly, fulfilling an obligation; (b) Look How Smart I Am: the desire is to impress with one's mathematical ability; (c) Check This Out: the desire is to obtain a payoff, which may be an intrinsic or extrinsic reward (Wigfield & Eccles, 2000; Zimmerman & Schunk, 2008); (d) I'm Really Into This: the desire is to experience the mathematical activity, entering flow (Csikszentmihalyi, 1990); (e) Don't Disrespect Me: the desire is to save face, meeting a challenge or threat to one's status or sense of well-being, as may occur in a highly charged discussion or argument; (f) Stay Out Of Trouble: the desire is to avoid possible conflict, distress, or embarrassment; (g) It's Not Fair: the desire is to correct an inequity; (h) Let Me Teach You: the desire is to assist someone else to understand the mathematics or solve the problem; (i) Pseudo-Engagement: the desire is to look good by appearing engaged, but avoid real participation. Instances of structures' activation are inferred from coded videotapes, while confirmation of motivating desires, behavior, and accompanying emotions comes from retrospective interviews with students and from questionnaire responses.

Limitations

The predominantly qualitative work focusing on state emotions, exemplified in these studies, suggests the desirability of far more complex descriptions of affective architecture in the study of emotion in mathematics education. But that very complexity, a consistent theme, points also to a degree of unpredictability in students' emotions. Some features of the psychological and social contexts influencing the inferred mathematical emotions are likely to be unknown and possibly unknowable. Replication of classroom situations where emotions occur is difficult to achieve and rarely attempted. The question of how reliably one can infer emotions from observations, especially complicated, subtle, or partially suppressed emotions, remains open—even with apparently corroborative questionnaire data and/or retrospective interview data. Findings tend to

be anecdotal, so that we do not know how generalizable they may be to other contexts or wider populations, and we cannot easily distinguish any that are spurious.

FUTURE DIRECTIONS: TOWARD THE UNIFICATION OF RESEARCH PERSPECTIVES

Quantitative studies can measure population characteristics and correlations, testing structural relations among trait emotions or between them and other easily quantifiable variables, and investigating whether a mathematical emotion is more often about mathematics or about something else (such as testing). But they leave out or average over the psychosocial contexts for emotions and disregard important possibilities of positive feelings about negative emotions (see further on). Fine-grained qualitative studies point to such complexities, but are small scale and labor intensive with high scale-up costs. Each such study illuminates at best a particular aspect of emotion in a specific mathematical context. And the methods of observation tend to influence researchers' theoretical perspectives profoundly. In my view, models based on still deeper ideas are needed: constructs sufficiently sophisticated to be able to address the domain-specific issues pertinent to mathematics, taking account of complex, situation-dependent interactions, yet at the same time providing a framework for generalizable descriptions of population characteristics, and offering systematic, research-based ways to improve the affective side of mathematics instruction through teachers' professional development.

In the balance of this chapter, I want to highlight four ideas that have been essential to the approach my collaborators and I have taken, which in my view deserve increased attention in mathematics education research: (a) the *representational* function of emotions, (b) emotions as functional components of *affective structures*, (c) the importance of *meta-affect*, and (d) the development of *mathematically powerful* affect. These pertain to both state and trait emotions.

Emotion as Representational

Emotional states continually encode and exchange information with cognitive systems of internal representation (Rogers, 1983; Zajonc, 1980). Emotions also serve *communicative* functions—sharing information and providing feedback among people in social situations. The *semantic content* of emotional feelings is implicit in constructs such as achievement emotions and essential to understanding the role state and trait emotions play in mathematical activity. For instance, during mathematical problem solving, *curiosity* may encode the possibility of new learning, evoking exploratory strategies for overcoming impasse. *Frustration* may serve to encode repeated failure of a strategy; as this emotion reaches a certain threshold strength, it can serve as a cue to the problem solver to try a different approach. The information encoded or exchanged through emotional expression may be about mathematical objects, the people engaged in mathematical discourse, the context of the activity, or where the problem solver stands in relation to expectations. Shared or interacting emotions carry information associated with mathematical cooperation, competition, or group processes during learning and problem solving.

State emotions typically encode complex information regarding the state of the learner or problem solver in relation to the problem environment—how likely one is to be able to learn something new or solve the problem in a reasonable length of time (*confidence*,

enthusiasm), how others see the problem solver (*pride, exuberance, bashfulness, shame*), the possibility of failure, including public failure (*apprehension, anxiety*), absence of the possibility of new learning or stimulating experience (*boredom*), the effectiveness of a team effort (*security, satisfaction*), how one measures up to another's success (*jealousy*), and so forth. All of these interpretations are *context-dependent*—an emotion such as satisfaction in different contexts may signify problem solving success, fulfillment of the desire to be acknowledged by others, success in conveying a mathematical concept or problem solution to another student, a sense that hard work has paid off, the failure of someone else who was envied or resented, or having gotten away with pretending to be engaged while doing something else.

Similarly, trait emotions encode longer-term information that is drawn on in mathematical situations. Both state and trait emotions carry complex meanings only minimally captured by their valence. And an emotion's importance is not necessarily proportional to its intensity.

Emotions Within Affective Structures

A promising theoretical direction is the characterization of *affective structures* with which trait emotions are interwoven and with which state emotions interact in characteristic ways. These are components of architecture whose specifics differ from person to person but for which some features are more-or-less invariant. Domain-specific structures involving emotion include mathematical self-concept and identity, mathematical intimacy (a valued, personally vulnerable, emotional relation between an individual and mathematics), mathematical integrity (a psychological posture valuing understanding and honesty in one's relation to mathematics) and their interactions (DeBellis & Goldin, 1997, 1999, 2006), sociocultural norms (Grouws & Lembke, 1996), and systems of beliefs and values in relation to mathematics (Goldin, 2002; Philipp, 2007). Some structures may be fundamentally relational with other people—for example, Hackenberg (2010) provides detailed qualitative analyses of *mathematical caring relations* she establishes with two students. Such structures may be deemed “high level,” describing global features of personality or relationship. The engagement structures discussed are termed “mid-level.” Their role is analogous to that of cognitive structures, such as proportional reasoning, which influence sequences of problem-solving steps or interactions. Just as describing cognitive structures and schemas has helped us to interpret both students' immediate problem solving and longer term mathematical understanding, characterizing affective structures can help us to interpret both students' state and trait emotions. This research direction suggests ways to address calls for extending the theory of motivation (Middleton & Spanias, 1999) and provides an alternative to considering cognitive, affective, and behavioral engagement as three distinct types of mathematical engagement (cf. Fredricks, Blumenfeld, & Paris, 2004).

Meta-Affect

The concept of *meta-affect*, in analogy with that of metacognition (Flavell, 1976), refers to affect about affect, affect about cognition about affect, and the monitoring and control of affect (DeBellis & Goldin, 1997, 2006; Goldin, 2002; Gómez-Chacón, 2000). With respect to emotions in mathematics education, meta-affect thus includes (context-dependent)

competencies pertaining to the control of the person's own emotional feelings, such as ways of coping with negative emotions—that is, what has been termed “meta-mood” and studied in the context of emotional intelligence (e.g., Fitness & Curtis, 2005). The “meta-emotional competencies” described by Op't Eynde et al. (2007) pertain to such self-regulation of emotion.

However, the idea of meta-affect involves far more than self-regulation—it incorporates the idea, familiar from everyday experience, that the experience of an emotion can be wholly transformed by the emotions one has about the emotion. For example, *fear* can be experienced with *elation*, as during a spectacular amusement park ride or a scary movie. *Pain* can be experienced with *joy*, as during strenuous physical activity. On the other hand, *pleasure* can be experienced *painfully*, *shamefully*, or *guiltily*, if it is undeserved, illicit, or lacks integrity. This kind of meta-affect is not typically voluntary, and as far as I know, has not been systematically investigated in the education literature.

During problem solving, the pivotal emotion of *frustration* may be experienced negatively (with the meta-affect of *apprehension*) when it encodes likely failure; we see this in qualitative studies of state anxiety. However, frustration can also be experienced positively (with the meta-affect of *anticipatory pleasure*) when it encodes the likelihood of the problem being intriguing—the solver, on becoming stuck, responding in effect, “This is a good one, don't tell me, I want to figure it out!” Likewise the *pleasure* of solving a problem correctly using a taught procedure can be experienced with discomfort, frustration, or even guilt when the solver does not understand an underlying concept. There can be many levels—conscious, preconscious, and unconscious—to such meta-affect. A student may describe *test anxiety* as about the immediate fear of failure under pressure. But behind it may lurk *pain*, the *shame* of acknowledging he did not earn the right to be *proud* in the face of expectations of a father whom he *loves*. The student is not consciously experiencing emotions of love or pride or shame in that moment, yet for this student in that context, the anxiety is about all of these other emotions—not simply about the test.

Meta-affect thus plays an essential role in the moment, so that emotions of either valence can be experienced positively or negatively. Both possibilities contribute to encoding strategic information. One may likewise conjecture that negative trait emotions toward mathematics can be experienced positively and contribute positively and vice versa. Such possibilities are typically averaged over, and thus unseen, in correlational studies.

Powerful Mathematical Affect

One explicit goal of research in mathematics education has been to characterize powerful problem-solving heuristics and strategies, insightful methods of visualization, and so forth. We need similarly far more detailed characterizations of *powerful affect* in mathematics—the emotional states and traits, the meta-affect, and the affective structures that enable one to ask questions, take the risk of being wrong, persevere in the face of impasse, create or engage with new representations, bring heuristic processes to bear, or plan anew. We need a new focus on how such powerful affect develops. And just as an explicit goal of research has been to characterize in detail mathematical misconceptions and how they may be corrected, we need a focus on how to intervene to correct disempowering affect—affect that interrupts concentration, enables avoidance, impedes understanding, or prevents its recognition when it occurs.

Feelings of negative valence—impatience, frustration, anxiety, or anger—occur during successful affective pathways, in expert problem solvers as well as students. It is a plausible conjecture that up to a point, negative emotions en route foster greater eventual pride, pleasure, and satisfaction in having attained a concept or solved a problem and that the experience of productive affective pathways in association with conceptually challenging mathematics contributes to the development of powerful global affective structures and trait emotions.

To sum up, there is much potential value in an integrated approach that draws on but distinguishes carefully the different characterizations of emotion and regards both state and trait emotion in the more sophisticated ways suggested here. The most immediate practical consequence, in my opinion, could be improved, research-based professional development of mathematics teachers that addresses the affective domain.

REFERENCES

- Akin, A., & Kurbanoglu, I. N. (2011). The relationships between math anxiety, math attitudes, and self-efficacy: A structural equation model. *Studia Psychologica*, *53*, 263–273.
- Alston, A., Goldin, G. A., Jones, J., McCulloch, A., Rossman, C., & Schmeelk, S. (2007). The complexity of affect in an urban mathematics classroom. In T. Lamberg & L. R. Wiest (Eds.), *Exploring mathematics education in context: Proceedings of the 29th annual meeting of PME-NA, Lake Tahoe, NV* (pp. 326–333). Reno, NV: University of Nevada.
- Apter, M. J. (Ed.). (2001) [cited in Lewis (2012)]. *Motivational styles in everyday life: A guide to reversal theory*. Washington, DC: American Psychological Association.
- Aronson, J., Lustina, M. J., Good, C., Keough, K., Steele, C. M., & Brown, J. (1999). When white men can't do math: Necessary and sufficient factors in stereotype threat. *Journal of Experimental Social Psychology*, *35*, 29–46.
- Artigue, M. (2002) [cited in McCulloch (2011)]. Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning*, *7*, 245–274.
- Baloğlu, M. (2002) [cited in Rayner et al. (2009)]. Construct and concurrent validity and internal consistency, split-half, and parallel-model reliability of the revised mathematics rating scale. *Dissertation Abstracts International*, *63*, 6-B (UMI No. 3058162).
- Baloğlu, M., & Koçak, R. (2006). A multivariate investigation of the differences in mathematics anxiety. *Personality and Individual Differences*, *40*, 1325–1335.
- Beasley, T. M., Long, J. D., & Natali, M. (2001). A confirmatory factor analysis of the mathematics anxiety scale for children. *Measurement and Evaluation in Counseling and Development*, *34*, 14–26.
- Bessant, K. C. (1995). Factors associated with types of mathematics anxiety in college students. *Journal for Research in Mathematics Education*, *26*(4), 327–345.
- Bowd, A. D., & Brady, P. H. (2003) [cited in Rayner et al. (2009)]. Gender differences in mathematics anxiety among preservice teachers and perceptions of their elementary and secondary school experience with mathematics. *Alberta Journal of Educational Research*, *49*, 24–36.
- Brady, P., & Bowd, A. (2005) [cited in Rayner et al. (2009)]. Mathematics anxiety, prior experience and confidence to teach mathematics among pre-service education students. *Teachers and Teaching: Theory and Practice*, *11*, 37–46.
- Cattell, R. B., & Scheier, I. H. (1961). *The meaning and measurement of neuroticism and anxiety*. New York, NY: Ronald Press.
- Chiu, L. H., & Henry, L. L. (1990). Development and validation of the Mathematics Anxiety Scale for Children. *Measurement and Evaluation in Counseling and Development*, *23*, 121–127.
- Csikszentmihalyi, M. (1990). *Flow: The psychology of optimal experience*. New York, NY: Harper & Row.
- Davis, R. B. (1984). *Learning mathematics: The cognitive science approach to mathematics education*. Norwood, NJ: Ablex.
- DeBellis, V. A., & Goldin, G. A. (1993). Analysis of interactions between affect and cognition in elementary school children during problem solving. In J. R. Becker & B. J. Pence (Eds.), *Procs. of the 15th Annual Meeting of*

- PME-NA, Vol. 2* (pp. 56–62). San Jose, CA: Center for Mathematics and Computer Science Education, San Jose State University.
- DeBellis, V. A., & Goldin, G. A. (1997). The affective domain in mathematical problem solving. In E. Pehkonen (Ed.), *Procs. of the 21st Annual Meeting of PME, Vol. 2* (pp. 209–216). Lahti, Finland: Univ. of Helsinki.
- DeBellis, V. A., & Goldin, G. A. (1999). Aspects of affect: Mathematical intimacy, mathematical integrity. In O. Zaslavsky (Ed.), *Procs. of the 23rd Annual Meeting of PME, Vol. 2* (pp. 249–256). Haifa, Israel: Technion Printing Center.
- DeBellis, V. A., & Goldin, G. A. (2006). Affect and meta-affect in mathematical problem solving: A representational perspective. *Educational Studies in Mathematics*, 63, 131–147. doi: 1.1007/s10649-006-9026-4
- Devine, A., Fawcett, K., Szűcs, D., & Dowker, A. (2012). Gender differences in mathematics anxiety and the relation to mathematics performance while controlling for test anxiety. *Behavioral and Brain Functions*, 8, 33. Retrieved from www.behavioralandbrainfunctions.com/content/8/1/33
- D’Mello, S. D., & Graesser, A. C. (2014). Confusion. In R. Pekrun & L. Linnenbrink-Garcia (Eds.), *Handbook of emotions in education*. New York, NY: Taylor & Francis.
- Dweck, C. S. (2000). *Self-theories: Their role in motivation, personality, and development*. Philadelphia, PA: Taylor & Francis.
- Ekman, P. (1992). An argument for basic emotions. *Cognition and Emotion*, 6, 169–20.
- Ekman, P., & Friesen, W. V. (2003). *Unmasking the face: A guide to emotions from facial expressions*. Cambridge, MA: Malor Books.
- Else-Quest, N. M., Hyde, J. S., & Hejmadi, A. (2008) Mother and child emotions during mathematics homework. *Mathematical Thinking and Learning*, 10, 5–35.
- Epstein, Y. M., Schorr, R. Y., Goldin, G. A., Warner, L. B., Arias, C., Sanchez, L., Dunn, M., & Cain, T. R. (2007). Studying the affective/social dimension of an inner-city mathematics class. In T. Lamberg & L. R. Wiest (Eds.), *Exploring mathematics education in context: Proceedings of the 29th annual meeting of PME-NA, Lake Tahoe, NV* (pp. 649–656). Reno, NV: University of Nevada.
- Fennema, E., & Sherman, J. A. (1976). Fennema-Sherman mathematics attitude scales: Instruments designed to measure attitudes toward the learning of mathematics by females and males. *Journal for Research in Mathematics Education*, 7, 324–326.
- Firestone, W. A., Schorr, R. Y., & Monfils, L. F. (Eds.) (2004). *The ambiguity of teaching to the test*. Mahwah, NJ: Erlbaum.
- Fitness, J., & Curtis, M. (2005). Emotional intelligence and the Trait Meta-Mood Scale: Relationships with empathy, attributional complexity, self-control, and responses to interpersonal conflict. *E-Journal of Applied Psychology: Social Section*, 1, 50–62.
- Flavell, J. H. (1976). Metacognitive aspects of problem solving. In L. B. Resnick (Ed.), *The nature of intelligence* (pp. 231–236). Hillsdale, NJ: Erlbaum.
- Forsyth, D. R. (1986). An attributional analysis of students’ reactions to success and failure. In R. S. Feldman (Ed.), *The social psychology of education: Current research and theory* (pp. 17–38). Cambridge, United Kingdom: Cambridge University Press.
- Fredricks, J. A., Blumenfeld, P. C., & Paris, A. H. (2004). School engagement: Potential of the concept, state of the evidence. *Review of Educational Research*, 74, 59–109.
- Frenzel, A. C., Pekrun, R., & Goetz, T. (2007). Perceived learning environment and students’ emotional experiences: A multilevel analysis of mathematics classrooms. *Learning and Instruction*, 17, 478–493.
- Goetz, T., Frenzel, A. C., Pekrun, R., Hall, N. C., & Lüdtke, O. (2007). Between- and within-domain relations of students’ academic emotions. *Journal of Educational Psychology*, 99, 715–733.
- Goldin, G. A. (2000). Affective pathways and representation in mathematical problem solving. *Mathematical Thinking and Learning*, 2, 209–219.
- Goldin, G. A. (2002). Affect, meta-affect, and mathematical belief structures. In G. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 59–72). Dordrecht, Netherlands: Kluwer.
- Goldin, G. A., Epstein, Y. M., Schorr, R. Y., & Warner, L. B. (2011). Beliefs and engagement structures: Behind the affective dimension of mathematical learning. *ZDM Mathematics Education*, 43, 547–56. doi: 1.1007/s11858-011-0348-z
- Goldin, G. A., Rösken, B., & Törner, G. (2009). Beliefs—no longer a hidden variable in mathematics teaching and learning processes. In J. Maasz & W. Schläglmann (Eds.), *Beliefs and attitudes in mathematics education: New research results* (pp. 1–18). Rotterdam, Netherlands: Sense.

- Gómez-Chacón, I. M. (2000). Affective influences in the knowledge of mathematics. *Educational Studies in Mathematics*, 43, 149–168.
- Grouws, D. A., & Lembke, L. O. (1996). Influential factors in student motivation to learn mathematics: The teacher and classroom culture. In M. Carr (Ed.), *Motivation in mathematics* (pp. 39–62). Cresskill, NJ: Hampton Press.
- Hackenberg, A. J. (2010). Mathematical caring relations in action. *Journal for Research in Mathematics Education*, 41, 236–273.
- Handal, B. (2003). Teachers' mathematical beliefs: A review. *The Mathematics Educator*, 13, 47–57.
- Hannula, M. S. (2002). Attitude towards mathematics: Emotions, expectations and values. *Educational Studies in Mathematics*, 49, 25–46.
- Hannula, M. S. (2006). Motivation in mathematics: Goals reflected in emotions. *Educational Studies in Mathematics*, 63, 165–178.
- Harper, N. W., & Daane, C. J. (1998) [cited in Rayner et al. (2009)]. Causes and reduction of math anxiety in pre-service elementary teachers. *Action in Teacher Education*, 19(4), 29–38.
- Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for Research in Mathematics Education*, 21, 33–46.
- Heyd-Metzuyanim, E., & Sfard, A. (2012). Identity struggles in the mathematics classroom: On learning mathematics as an interplay of mathematizing and identifying. *International Journal of Educational Research*, 51–52, 128–145.
- Ho, H.-Z., Senturk, D., Lam, A. G., Zimmer, J. M., Hong, S., & Okamoto, Y. (2000). The affective and cognitive dimensions of math anxiety: A cross-national study. *Journal for Research in Mathematics Education*, 31, 362–379.
- Hopko, D. R., Mahadevan, R., Bare, R. L., & Hunt, M. K. (2003) [cited in Devine et al. (2012)]. The abbreviated math anxiety scale (AMAS): Construction, validity, and reliability. *Assessment*, 10, 178–182.
- Izard, C. E. (1983 revised) [cited in DeBellis & Goldin (1993)]. *The maximally discriminative facial movement coding system*. Newark, DE: University of Delaware, Instructional Resources Center.
- Izard, C. E. (1991). *The psychology of emotions*. New York, NY: Plenum.
- Jain, S., & Dowson, M. (2009). Mathematics anxiety as a function of multidimensional self-regulation and self-efficacy. *Contemporary Educational Psychology*, 34, 240–249.
- Karsenty, R. (2004). Mathematical self-schema: A framework for analyzing adults' retrospection on high school mathematics. *Journal of Mathematical Behavior*, 23, 325–349.
- Kelly, A. E., & Lesh, R. A. (Eds.). (2000). *Handbook of research design in mathematics and science education*. Mahwah, NJ: Erlbaum.
- Laurent, J., Catanzaro, S. J., Joiner, T. E., Jr., Rudolph, K. D., Potter, K. I., & Lambert, S. (1999). A measure of positive and negative affect for children: Scale development and preliminary validation. *Psychological Assessment*, 11, 326–338.
- Leder, G. C., Pehkonen, E., & Törner, G. (Eds.). (2002). *Beliefs: A hidden variable in mathematics education?* Dordrecht, Netherlands: Kluwer.
- Lee, J. (2009). Universals and specifics of math self-concept, math self-efficacy, and math anxiety across 41 PISA 2003 participating countries. *Learning and Individual Differences*, 19, 355–365.
- Lesh, R. A., & Lamon, S. J. (Eds.) (1992). *Assessment of authentic performance in school mathematics*. Washington, DC: American Association for the Advancement of Science.
- Lesh, R. A., & Landau, M. (Eds.) (1983). *Acquisition of mathematics concepts and processes*. New York, NY: Academic Press.
- Lewis, G. (2011). Mixed methods in studying the voice of disaffection with school mathematics. *Proceedings of the British Society for Research into Learning Mathematics*, 31(3), 101–106.
- Lewis, G. (2012). A portrait of disaffection with school mathematics: The case of Helen. In T. Y. Tso (Ed.), *Proceedings of the 36th conference of the International Group for the Psychology of Mathematics Education* (Vol. 3, pp. 115–122). Taipei, Taiwan: PME.
- Linnenbrink, E. A., & Pintrich, P. R. (2004). Role of affect in cognitive processing in academic contexts. In D. Y. Dai & R. J. Sternberg (Eds.), *Motivation, emotion, and cognition: Integrative perspectives on intellectual functioning and development* (pp. 57–87). Mahwah, NJ: Erlbaum.
- Ma, X. (1999). A meta-analysis of the relationship between anxiety toward mathematics and achievement in mathematics. *Journal for Research in Mathematics Education*, 30, 520–54.
- Ma, X., & Xu, J. (2004). The causal ordering of mathematics anxiety and mathematics achievement: A longitudinal panel analysis. *Journal of Adolescence*, 27, 165–18.

- Maasz, J., & Schlöglmann, W. (Eds.). (2009). *Beliefs and attitudes in mathematics education: New research results*. Rotterdam, Netherlands: Sense.
- Malmivuori, M.-L. (2006). Affect and self-regulation. *Educational Studies in Mathematics*, 63, 149–164.
- Maloney, E. A., & Beilock, S. L. (2012). Math anxiety: Who has it, why it develops, and how to guard against it. *Trends in Cognitive Sciences*, 16(8), 404–406.
- McCulloch, A. W. (2011). Affect and graphing calculator use. *Journal of Mathematical Behavior*, 30, 166–179.
- McLeod, D. B. (1987) [cited in Zan et al. (2006)]. A constructivist approach to research on attitude toward mathematics. In J. Bergeron, N. Herscovics, & C. Kieran (Eds.), *Proceedings of the 11th PME, Montreal*, Vol.1, pp.133–139. ERIC Clearinghouse (eric.ed.gov), ED 383 532
- McLeod, D. B. (1989). Beliefs, attitudes and emotions: New views of affect in mathematics education. In D. McLeod and V. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 245–258). New York, NY: Springer.
- McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. Grouws (Ed.), *Handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 575–596). New York, NY: Macmillan.
- McLeod, D. B. (1994). Research on affect and mathematics learning. *Journal for Research in Mathematics Education*, 25, 637–647.
- Meece, J. L., Wigfield, A., & Eccles, J. (1990). Predictors of math anxiety and its influence on young adolescents' course enrollment intentions and performance in mathematics. *Journal of Educational Psychology*, 82, 60–7.
- Middleton, J. A., & Spanias, P. A. (1999). Motivation for achievement in mathematics: Findings, generalizations, and criticisms of the research. *Journal for Research in Mathematics Education*, 30(1), 65–88.
- Mora, R. (2011). "School is so boring": High-stakes testing and boredom at an urban middle school. *Penn GSE Perspectives on Urban Education*, 9(1), www.urbanedjournal.org
- Mullis, I.V.S., Martin, M. O., & Foy, P. (with Olson, J. F., Preuschoff, C., Erberber, E., Arora, A., & Galia, J.) (2008). *TIMMS (Trends in International Mathematics and Science Study) 2007 International Mathematics Report*, Chapter 4. Chestnut Hill, MA: TIMMS & PIRLS International Study Center, Boston College.
- Nardi, E., & Steward, S. (2003). Is mathematics T.I.R.E.D.? A profile of quiet disaffection in the secondary mathematics classroom. *British Educational Research Journal*, 29(3), 345–367.
- OECD [Organization for Economic Co-operation and Development] (2004). Student learning: Attitudes, Engagement, and Strategies. In *Learning for Tomorrow's World – First Results from PISA 2003* (pp. 109–158). Retrieved from www.oecd.org/edu/school/programmeforinternationalstudentassessmentpisa/learningfortomorrowsworldfirstresultsfrompisa2003.htm
- Op't Eynde, P., De Corte, E., & Verschaffel, L. (2006). Accepting emotional complexity: A socioconstructivist perspective on the role of emotions in the mathematics classroom. *Educational Studies in Mathematics*, 63, 193–207.
- Op't Eynde, P., De Corte, E., & Verschaffel, L. (2007). Students' emotions: A key component of self-regulated learning? In P. Schutz & R. Pekrun (Eds.), *Emotion in education* (pp. 185–204). Burlington, MA: Academic Press.
- Op't Eynde, P., & Hannula, M. S. (2006). The case study of Frank. *Educational Studies in Mathematics*, 63, 123–129.
- Ortony, A., Clore, G. L., & Collins, A. (1988). *The cognitive structure of emotions*. Cambridge, United Kingdom: Cambridge University Press.
- Pekrun, R. (2006). The control-value theory of achievement emotions: Assumptions, corollaries, and implications for educational research and practice. *Educational Psychology Review*, 18, 315–341.
- Pekrun, R., Goetz, T., Frenzel, A. C., Barchfeld, P., & Perry, R. P. (2011). Measuring emotions in students' learning and performance: The Achievement Emotions Questionnaire (AEQ). *Contemporary Educational Psychology*, 36, 36–48.
- Pekrun, R., & Linnenbrink-Garcia, L. (2012). Academic emotions and student engagement. In S. L. Christenson, A. L. Reschly, & C. Wylie (Eds.), *Handbook of research on student engagement* (pp. 259–282). New York, NY: Springer. doi: 1.1007/978-1-4614-2018-7_12
- Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 257–318). Charlotte, NC: Information Age Publishing.
- Plake, B. S., & Parker, C. S. (1982). The development and validation of a revised version of the Mathematics Anxiety Rating Scale. *Educational and Psychological Measurement*, 42, 551–557.

- Rayner, V., Pitsolantis, N., & Osana, H. (2009). Mathematics anxiety in preservice teachers: Its relationship to their conceptual and procedural knowledge of fractions. *Mathematics Education Research Journal*, 21(3), 60–85.
- Richardson, F. C., & Suinn, R. M. (1972). The Mathematics Anxiety Rating Scale: Psychometric data. *Journal of Counseling Psychology*, 19, 551–554.
- Rogers, T. B. (1983). Emotion, imagery, and verbal codes: A closer look at an increasingly complex interaction. In J. Yuille (Ed.), *Imagery, memory, and cognition: Essays in honor of Alan Paivio* (pp. 285–305). Hillsdale, NJ: Erlbaum.
- Schiefele, U., & Csikszentmihalyi, M. (1995). Motivation and ability as factors in mathematics experience and achievement. *Journal for Research in Mathematics Education*, 26, 163–181.
- Schoenfeld, A. H. (1985). *Mathematical problem solving*. New York, NY: Academic Press.
- Schorr, R. Y., Epstein, Y. M., Warner, L. B., & Arias, C. C. (2010a). Don't disrespect me: Affect in an urban math class. In R. Lesh, P. L. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling students' mathematical modeling competencies: ICTMA 13* (pp. 313–325). New York, NY: Springer.
- Schorr, R. Y., Epstein, Y. M., Warner, L. B., & Arias, C. C. (2010b). Mathematical truth and social consequences: The intersection of affect and cognition in a middle school classroom. *Mediterranean Journal for Research in Mathematics Education*, 9, 107–134.
- Schorr, R. Y., & Goldin, G. A. (2008). Students' expression of affect in an inner-city Simcalc classroom. *Educational Studies in Mathematics*, 68(2), 131–148.
- Sherman, B. F., & Wither, D. P. (2003). Mathematics anxiety and mathematics achievement. *Mathematics Education Research Journal*, 15(2), 138–15.
- Shuman, V., & Scherer, K. R. (2014). Concepts and structure of emotions. In R. Pekrun & L. Linnenbrink-Garcia (Eds.), *Handbook of emotions in education*. New York, NY: Taylor & Francis.
- Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching*, 77, 20–26.
- Skinner, E., Kindermann, T., & Furrer, C. (2009). A motivational perspective on engagement and disaffection: Conceptualization and assessment of children's behavioral and emotional participation in academic activities in the classroom. *Educational and Psychological Measurement*, 69, 493–525.
- Smith, J. P. III. (1996). Efficacy and teaching mathematics by telling: A challenge for reform. *Journal for Research in Mathematics Education*, 27(4), 387–402.
- Spielberger, C. D., Edwards, C. D., Montuori, J., & Lushene, R. (1973). *State-trait anxiety inventory for children*. Palo Alto, CA: Mind Garden.
- Sriraman, B., & English, L. (Eds.) (2010). *Theories of mathematics education: Seeking new frontiers*. New York, NY: Springer.
- Steele, C. M., & Aronson, J. (1995). Stereotype threat and the intellectual test performance of African Americans. *Journal of Personality and Social Psychology*, 69, 797–811.
- Steffe, L., Neshet, P., Cobb, P., Goldin, G. A., & Greer, B. (Eds.) (1996). *Theories of mathematical learning*. Hillsdale, NJ: Erlbaum.
- Suinn, R. M. (1988) [cited in Beasley et al. (2001)]. *Mathematics Anxiety Rating Scale—E (MARS-E)*. Fort Collins, CO: Rocky Mountain Behavioral Science Institute.
- Tobias, S. (1993). *Overcoming math anxiety*. New York, NY: W. W. Norton & Company.
- Trujillo, K. M., & Hadfield, O. D. (1999) [cited in Rayner et al. (2009)]. Tracing the roots of mathematics anxiety through in-depth interviews with preservice teachers. *College Student Journal*, 33(2), 219–232.
- Uusimäki, L., & Nason, R. (2004) [cited in Rayner et al. (2009)]. Causes underlying pre-service teachers' negative beliefs and anxieties about mathematics. In M. J. Hoines & A. B. Fuglestad (Eds.), *Proceedings of the 28th conference of the International Group for the Psychology of Mathematics Education* (Vol. 4, pp. 369–376). Bergen, Norway: Bergen University.
- Walen, S. B., & Williams, S. R. (2002). A matter of time: Emotional responses to timed mathematics tests. *Educational Studies in Mathematics*, 49, 361–378.
- Wellborn, J. G. (1991) [cited in Skinner et al. (2009)]. *Engaged and disaffected action: The conceptualization and measurement of motivation in the academic domain*. Unpublished doctoral dissertation, University of Rochester, NY.
- Widmer, C. C., & Chavez, A. (1982) [cited in Rayner et al. (2009)]. Math anxiety and elementary school teachers. *Education*, 102(3), 272–276.

- Wigfield, A., & Eccles, J. S. (2000). Expectancy-value theory of achievement motivation. *Contemporary Educational Psychology, 25*, 68–81.
- Wigfield, A., & Meece, J. L. (1988). Math anxiety in elementary and secondary school students. *Journal of Educational Psychology, 80*, 210–216.
- Zajonc, R. B. (1980). Feeling and thinking: Preferences need no inferences. *American Psychologist, 35*, 151–175.
- Zan, R., Brown, L., Evans, J., & Hannula, M. S. (2006). Affect in mathematics education: An introduction. *Educational Studies in Mathematics, 63*(2), 113–121.
- Zeidner, M. (2014). Anxiety in education. In R. Pekrun & L. Linnenbrink-Garcia (Eds.), *Handbook of emotions in education*. New York, NY: Taylor & Francis.
- Zimmerman, B. J., & Schunk, D. H. (2008). Motivation: An essential dimension of self-regulated learning. In D. H. Schunk & B. J. Zimmerman (Eds.), *Motivation and self-regulated learning: Theory, research, and applications* (pp. 1–30). New York, NY: Routledge.