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ORIGINAL ARTICLE

Beliefs and engagement structures: behind the affective dimension of mathematical learning

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Abstract Beliefs influencing students' mathematical learning and problem solving are structured and intertwined with larger affective and cognitive structures. This theoretical article explores a psychological concept we term an *engagement structure*, with which beliefs are intertwined. Engagement structures are idealized, hypothetical constructs, analogous in many ways to cognitive structures. They describe complex "in the moment" affective and social interactions as students work on conceptually challenging mathematics. We present engagement structures in a self-contained way, paying special attention to their theoretical justification and relation to other constructs. We suggest how beliefs are characteristically woven into their fabric and influence their activation. The research is based on continuing studies of middle school students in inner-city classrooms in the USA.

1 Context of the research

Conceptually challenging mathematics involves gaining or changing some understanding. It usually entails nonroutine thinking, where mathematical meanings matter more than procedures, or problem solving with experiences of impasse and construction of new representations. Its classroom context includes social activity such as discussion and argument, and exploration which can be solitary or social. As individual students share ideas, there occur wrong answers, blind alleys, and fruitful suggestions. In such situations, students often disagree and criticize each others' ideas. Social interactions may evoke strong emotional feelings, leading sometimes to deeper engagement and other times to disaffection.

Picture an urban middle school classroom in a low-income, predominantly minority US community. Early adolescent children (ages 12–14 years) work in small groups on a challenging math problem. Their teacher, having presented the task, moves from group to group providing encouragement and asking occasional questions, a frequent practice of this teacher, though far from standard in many US schools. Some children work individually, deeply engaged in the math. Some talk with others about the problem or about other things. Some appear distracted or bored, others stuck or confused, and a few express frustration. One student tries to explain the problem to another who does not understand it. Sometimes, one student disagrees with another; occasionally, offense is taken. One or two groups think they have solved the problem. A student expresses disappointment that it was too easy.

As such affective/social interactions occur, students grapple with the underlying problem structures that involve additive, multiplicative or recursive processes. Their cognitions vary widely. Individuals respond differently to the teacher's directions, have different prior knowledge, construct different understandings, and propose different problem representations and strategies. They bear distinct and changing mathematical conceptions and misconceptions, and engage differently with ideas proposed by others.

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Students hold a spectrum of beliefs—about the school setting and expectations, their peers and their standing with their peers, the teacher and her expectations, mathematics and what it means to learn it, parental expectations, their own abilities, etc. In complex ways, such beliefs influence their social interactions, problem solving, and in-the-moment engagement. Other kinds of more enduring personal orientations—e.g., attitudes, values, achievement orientations, and personality traits—also affect how they frame their classroom experiences.

How can one best *understand usefully* such complex influences on students' cognitive, affective, and/or behavioral engagement with mathematics, i.e., the *why* of the dynamics? Taking engagement to be a key affordance of meaningful learning, how can one identify and study teaching strategies that deepens it—in the specific socio-economic setting studied here, and across different venues?

This article proposes a theoretical concept we call *engagement structures* to help account for recurring, dynamical patterns of interaction around mathematics. Briefly, an engagement structure is an *idealization* involving a characteristic motivating desire or goal, actions including social behaviors toward fulfilling the desire, supporting beliefs, "self-talk," sequences of emotional states, meta-affect, strategies, and possible outcomes—a kind of *behavioral/affective/social constellation* situated in the person, becoming active in social contexts. We discuss examples to which we give the following names: *Get The Job Done, Look How Smart I Am, Don't Disrespect Me, Check This Out, I'm Really Into This, Let Me Teach You, It's Not Fair, Stay Out of Trouble*, and *Pseudo-Engagement*.

The paper is organized as follows. Section 2 presents the ideas in a self-contained way, and justifies theoretically their value to research in the psychology of mathematical engagement. We discuss theoretical issues around the engagement structure construct, describing its hypothesized components and how they are conjectured to interact. There follow more specific descriptions of nine proposed structures. In place of a general review of related research, we consider as we proceed various connections and contrasts between the components of engagement structures and related ideas in the affect and motivation literature. Section 3 elaborates on the desirability of different engagement structures, stages in their activation, their specificity and universality, and the process of branching from one active structure to another. Section 4 then explores, in a preliminary fashion, the intimate relationship between engagement structures and beliefs characteristically woven into their fabric. Section 5 sums up, including potential implications, limitations, and future directions. As our present purpose is theoretical, we refer to other sources for descriptions of our ongoing empirical work (Alston et al., 2007; Epstein et al., 2007, 2010; Schorr & Goldin, 2008; Schorr et al., 2010a, b).

2 Engagement structures: theoretical perspectives

2.1 General considerations

Establishing the scientific value of a new construct requires addressing several criteria: its *need* and the purposes it serves; its *fit* with other proven constructs; its *theoretical utility* in characterizing or explaining phenomena under discussion; the potential for its *empirical validation* or confirmation; its *empirical utility*, e.g., in suggesting *new questions* for study; and importantly, its potential *practical value*—here, as a tool to improve mathematics teaching.

The construct of *beliefs* (the central focus of the present volume) took a long time to be recognized as important (Leder et al., 2002; Maass & Schlöglmann, 2009), but in our view would satisfy these criteria. We suggest that many or most of the criteria can be met by the engagement structure construct, which we seek to situate in the wider context of related research.

2.2 Level of description

The literature describes some "high level" constructs that capture important aspects of affect and suggest some of the particulars of interactions among beliefs, emotional feelings, classroom social interactions, and mathematics learning. These include mathematical identity, self-concept, and self-efficacy (e.g., Pajares & Graham, 1999), centered more in the psychology of individual learners, and sociocultural norms (e.g., Grouws & Lembke, 1996), centered more in the social context of mathematical learning.

In the study of cognition, we have long had available what might be called "mid-level" constructs that help describe and explain situated mental representation and behavior—e.g., frames or interpretive schemes through which individuals filter their experiences of events (Goffman, 1974; Greeno & Goldman, 1988; see also van de Sande & Greeno, 2010), social scripts involving stereotyped patterns of situated discourse or event sequences (Abelson, 1981), and *specific* cognitive structures or *schemas* pertinent to mathematical understanding and problem solving; e.g., proportional reasoning, coordination of conditions, and so forth (e.g., Davis, 1984). Such structures become active during problem solving and to a certain extent "govern" the problem solver's in-the-moment cognitive activity.

The idealized constellations of in-the-moment desires, behavioral patterns, pathways of emotion, and other strands that we call engagement structures are proposed at an analogous level of complexity to cognitive structures. *Constructs at such an intermediate level are needed, centered in the affective domain, to similarly help us understand and explain students' mathematical engagement*. Thus, an engagement structure (a) is substantially *more* elaborate and complex than a goal, a pattern of behavior, or an emotional state; (b) can be identified through *patterns that repeat* or occur commonly in an individual, as opposed to one-time patterns or stories specific to an occasion; (c) can be identified as *widely or near-universally present* in many different people, so that its activation is *possible for most or all students* in pertinent social situations; and (d) is *recognizable* and potentially *helpful to teachers of mathematics*.

The *theoretical utility* of this level of description is that one can "point to," potentially measure, and discuss easily multiple complex patterns that may be useful to researchers and seem to resonate with teachers' experience and judgment. The description of an engagement structure goes beyond characterizing instances of mathematical engagement as behavioral, cognitive, and/or affective (cf. Fredricks, Blumenfeld, & Paris, 2004, and references therein), or the classification of a student as very engaged, partially engaged, or disengaged. For example, an engagement structure such as *Let Me Teach You* suggests a very different dynamic from that of *Look How Smart I Am*, although both describe students engaged with communicating mathematical ideas, and both explicitly incorporate affective, cognitive, and behavioral components (as described below).

Our level of description makes it possible to consider specific ways in which mathematical *beliefs* can interact with a student's in-the-moment engagement in a social/mathematical situation (see Sect. 4).

We do *not* propose at this stage to "reify" engagement structures, to take them as established or given, or to imply the existence of identifiable neurological structures. Rather, they are *hypothetical* psychological constructs internal to an individual that can become active under specific social conditions. They describe characteristic, identifiable patterns of emotional feelings, goals, and behaviors (including social interactions) that "govern" a student's in-the-moment engagement for minutes (or even seconds) at a time during a class period.

The skeptical reader, without adopting our underlying theoretical perspective, may consider this article as offering an observation-based catalog of some typical ways in which urban mathematics students in low-income, predominantly minority communities "frame" their day-to-day school experiences. One may then judge the usefulness of the categorization to educators and/or propose different theoretical underpinnings for the patterns described. Evaluation of competing theoretical understandings rests ultimately on experimental findings that can distinguish them.

2.3 Interacting strands in engagement structures

The term *structure* suggests components functioning not in isolation but in interaction, to form a coherent construct that is "more than the sum of its parts." The components or strands are regarded as *simultaneously present* and *dynamically interacting*. Their nature leads us to characterize an engagement structure as a "behavioral/affective/social constellation." We propose ten such strands (cf. Goldin, Epstein, & Schorr, 2007). The first seven describe in-the-moment, changing or changeable aspects of the student's state; the latter three pertain to interactions with longer-term attributes or more "global" structures:

(1) a characteristic *goal* or *motivating desire*,
(2) characteristic *patterns of behavior* including social interactions oriented toward fulfilling the desire,
(3) a characteristic *affective pathway* experienced by the individual,
(4) external *expressions of affect*,
(5) *meanings* encoded by emotional feelings,
(6) *meta-affect* pertaining to emotional states,
(7) characteristic *self-talk* or inner speech,
(8) interactions with *systems of beliefs and values*,
(9) interactions with longer-term *traits*, *characteristics*, and *orientations*, and
(10) interactions with characteristic problem-solving *strategies* and *heuristics*.

Many of these strands are affective. McLeod (1994) describes the affective domain in mathematics education as including emotions (rapidly changing, transitory, highly affective), attitudes (more stable, more cognitive), and beliefs (most enduring, highly cognitive as well as affective). DeBellis & Goldin (2006) take values (including ethics and morals) as a distinct affective component (cf. Rokeach, 1968, 1973). Schoenfeld (2010) uses the broader term "orientations" to include "beliefs, values, biases, dispositions, etc." (p. viii). Most or all of these enter our description, as we consider each proposed strand briefly.

(1) Having a *goal* suggests an internal, cognitive representation, while the term *motivating desire* is intended to suggest *affect*—discomfort with non-fulfillment, anticipated satisfaction, pleasure, or release of emotion in fulfillment. The specific desire is evoked by the social environment as it is construed. It occurs as the student senses an opportunity to fulfill a manifest or latent *need* (Murray, 2008, 70th Anniv. Edition) by pursuing a concrete, immediate goal. Motivating desires are more situation specific than needs. The *environmental press* (Murray's term for situational constraints facing the person) interacts with the need, impelling *actions* toward fulfilling the desire.

The importance of goals and their interaction with affect is uncontroversial. Middleton & Spanias (1999) discuss goal theory extensively in the context of mathematics learning, including the interplay of goals with beliefs and motivation. Schoenfeld (2010) bases teachers' "in the moment" decision-making on three components: goals, resources, and orientations. In the psychology of personality, one distinguishes *traits* (longer-term, stable characteristics) from *states* (rapidly changing particulars that influence behavior "in the moment") (Cattell & Scheier, 1961). Thus, we here distinguish goals or goal orientations that are longer-term characteristics of individuals from a goal actively held on a particular occasion. Motivational features such as achievement goal orientations are often understood as "trait-like" (e.g., Schutz & Pekrun, 2007; Schunk & Zimmerman, 2008).

Anderman and Wolters (2006) review some now well-established classifications, including *approach* versus *avoidance* goals and *mastery* versus *performance* goals; e.g., "mastery-approach goals" (Pintrich, 2000) involve understanding the material, while "performance-approach goals" involve achieving more than others. Much research then seeks to attribute learning effects to the types of goals held by students; e.g., Anderman and Wolters (2006, p. 371) write, "Although there is much debate concerning the effects of performance-approach goals, there is general consensus that performance-avoid goals are maladaptive and related negatively to many valued educational outcomes." Linnenbrink (2007, p. 122) proposes "…a triarchic model of reciprocal causations…in which there are reciprocal relations among achievement goal orientations and affect, affect and engagement, and achievement goal orientations and engagement."

Much research suggests student engagement correlates with academic achievement (Finn, 1993; Greenwood, 1991; Marks, 2000). Park (2005) finds positive effects for engagement on students' mathematical growth, independent of socioeconomic or minority status. To perform such studies, *engagement* must be interpreted as more enduring than an in-the-moment state, so one typically attributes to students (trait-like) degrees of engagement over time.

While acknowledging the importance of longer-term goal orientations and their relationship to learning mathematics, we highlight here the *great variability* of *in-the-moment* goals. Some research on goal orientations notes their context dependence, moving a step toward our perspective. Some of the specific motivating desires described here can be understood as "counterparts" to longer-term goal orientations: thus, the motivating desire for *I'm Really Into This* seems more like a mastery-approach goal; that for *Look How Smart I Am* like a performance-approach goal; and that for *Stay Out of Trouble* like a performance-avoid goal. But other motivating desires relate more directly to personality theory, or to theories of social interactions. Goal orientations as longer-term student traits are likely to influence the *thresholds* for activation of particular motivating desires in various circumstances, but (in our view) are unlikely to rule in or rule out any of them deterministically; different motivating desires become active, in the same individual, under different social conditions.

(2) Characteristic *patterns of behavior* are oriented toward fulfilling the motivating desire. Social interactions, behavioral outcomes, and contingencies form part of the pattern. Different motivating desires may result in similar behaviors—a student wanting to impress the teacher, one trying simply to complete the assignment, one seeking to master underlying mathematical ideas, or one pursuing a promised reward may all pay attention, engage in discussion, and put pencil to paper and work toward solving an assigned problem. Fredricks et al. (2004) discuss behavioral, cognitive, and affective engagement and their overlaps. Our perspective is that in-the-moment engaged behavior oriented toward *fulfilling a motivating desire* provides a structural link between our first two components, suggesting the close intertwining of affective and behavioral engagement.

(3) A characteristic *affective pathway* or *sequence of emotional feelings* experienced (internally) by the individual commences with arousal of the motivating desire. The relation between goals, behavior, and emotional feelings is an important research theme; e.g., Anderman and Wolters (2006) discuss the synthesis of goals, values (appraisals), and emotional responses:

> "Emotional responses can also be viewed as a consequence of goal setting and goal orientation. Target goals drive affective reactions by serving as standards or objectives that students use to evaluate their progress (Schunk, 2001). Students' appraisals of whether they have achieved or made sufficient progress toward their target goals have emotional implications (Linnenbrink & Pintrich, 2000). Students may feel joy, relief, or pride in accomplishment for the goals they have reached or are progressing toward at a reasonable rate. Alternatively, when students are focused on avoiding certain outcomes, the ability to move away from or create distance from these unwanted outcomes can also produce positive emotional reactions (Linnenbrink & Pintrich, 2002). In a similar way, negative emotional responses follow when progress toward goals is judged to be slow, insufficient, or absent altogether. In this way, having target or behavioral goals sets the stage for experiencing emotions within academic settings."
> (Anderman & Wolters, 2006, pp. 381–382)

Pekrun et al. (2007) consider "emotions tied directly to achievement activities or achievement outcomes," offering a taxonomy of such achievement emotions. They provide an overview of the "control-value theory" that predicts how patterns of appraisals lead to different achievement emotions.

Particular emotions are typically characterized as positive (e.g., joy, relief, pride, elation) or negative (e.g., boredom, anxiety, anger, frustration). In surveys of self-reported emotional feelings, positive emotions typically correlate with each other, as do negative emotions (e.g., Laurent et al., 1999). But this can conceal essential features. Patterns of affect associated with constructive engagement do not exclusively involve curiosity, excitement, fun, and satisfaction, but include feelings of impasse, frustration, and disappointment. When the emotional journey to mathematical success is arduous, even painfully so, the resulting satisfaction of achievement may be more profound. As mathematics educators, we must come to understand how "negative" feelings can and often do support engagement, persistence, and learning. Goldin (2000) describes idealized affective pathways during mathematical problem solving that interact with heuristics and can result in forming longer-term, "global" structures. Here, a feeling such as frustration can contribute positively or negatively, according to the pathway in which it occurs.

Emotions are often expressive of interactions between the *individual* and the *social*. Malmivuori (2006) discusses the self-regulatory function of affect in relation to mathematics learning and the social environment. Research on social environments in schools within low-income, minority urban communities (Anderson, 1999, Dance, 2002) describes "street culture" and its influence on the affective lives of schoolchildren, suggestive of the affective pathways associated with *Don't Disrespect Me* (involving emotions of resentment or anger, and pride) or *Stay Out of Trouble* (involving emotions of apprehension or fear, and relief).

Engagement structures entail characteristic emotions in connection with initial activation of a motivating desire, the ensuing behaviors and their in-the-moment social consequences, and the fulfillment or non-fulfillment of the desire.

(4) *Expressions from which affect may be inferred* are socioculturally dependent as well as idiosyncratic, and also serve communicative functions: emotionally expressive words, interjections and exclamations, eye contact, facial expressions, posture and "body language," hand and body movements including touching or gestures toward others, agitation, tears, laughter, blushing, etc. Such expressions help inform us of the student's motivating desire, the reasons for particular behavioral patterns, and the emotional feelings occurring, e.g., during *Let Me Teach You*, leaning forward so as to embrace another student's work, or *Don't Disrespect Me*, jumping up angrily to argue assertively.

(5) Building on the ideas of Zajonc (1980), Rogers (1983), and others, Goldin (2000) discusses affect as a *system of internal representation*, suggesting that emotional states *encode information* or *meanings* exchanged with verbal, imagistic, notational, and strategic representational systems during problem solving. Skinner et al. (2009), seeking to assess affective engagement through self-reports of positive emotional feelings, distinguish "engaged emotion" from "disaffected emotion." The former, as operationalized in their work (adapted from Wellborn, 1991), includes reported experiences of feeling "good" or "interested" in class, and experiencing class as "fun;" the latter includes feeling "bored," "nervous," "bad," "mad," or "frustrated" in class, experiencing class as "not all that fun for me." But from our perspective, the *meanings* associated with such emotions for students are highly context dependent. Feeling "good" can also signify the absence of challenge or difficulty, while "fun" can signify diverting social interactions. Likewise, "frustration" can signify serious *engagement with impasse*. Considering emotions and their meanings in the context of engagement structures hypothesized to be active, it is natural to *interpret* students' emotional feelings and discuss their *meanings* on the particular occasions of their occurrence.

(6) In analogy with metacognition (Flavell, 1979), *meta-affect* includes "affect about affect, affect about and within cognition that may again be about affect, monitoring of affect, and affect as monitoring." (Goldin, 2002, p. 59; see also DeBellis & Goldin, 2006). Meta-affect can transform the experience of emotion from negative to positive or the reverse: thus *fear* can be felt as extremely negative (e.g., severe fear of flying) or profoundly positive (e.g., the thrill of a scary amusement park ride); *frustration* with a mathematical task can likewise be experienced as negative (e.g., anticipation of failure) or positive (e.g., enhanced interest in the challenge). Meta-affect is especially important in connection with beliefs; Goldin (2002) suggests, "prevailing belief structures…are powerfully stabilized by meta-affect. Such beliefs are unlikely to change simply because factual warrants for alternate beliefs are offered." (p. 70).

Different engagement structures may evoke different meta-affective responses. For example, when *Get The Job Done* is active, frustration is likely to be experienced negatively, signifying barriers to fulfilling the motivating desire; in the context of *I'm Really Into This*, frustration is more likely to be experienced positively, signifying challenge and heightening intrinsic mathematical interest in the problem.

(7) *Self-talk* or inner speech, a concept from cognitive therapy research (Beck, 1976), occurs in response to and is evocative of the student's emotions, beliefs, and motivating desire. We hypothesize patterns of self-talk associated with the activation of each engagement structure, facilitating the internal organization of the structure. For example, "He has no right to talk to me that way," and "I'm not going to let him get away with it," may characterize activation of the structure *Don't Disrespect Me*.

(8) Interactions with *beliefs and values* are a central focus of this article. The book by Leder et al. (2002) offers various perspectives on beliefs. Op 't Eynde et al. (2002) distinguish students' beliefs about mathematics education, about themselves, and about the social context where they are learning. McLeod and McLeod (2002) review important consequences of mathematical beliefs and point to open questions, from early work on limitations beliefs impose during problem solving (e.g., Schoenfeld, 1985) to differences of definition among researchers. Goldin (2002, p. 59) takes beliefs to be "multiply-encoded, internal cognitive/affective configurations, to which the holder attributes truth value of some kind (e.g., empirical truth, validity, or applicability)." Pajares (1992) reviews the problem of defining beliefs, which continues unresolved (cf. Furinghetti & Pehkonen, 2002; Törner, 2002).

Certain specific, widespread beliefs influence mathematical motivation and learning. Kloosterman (1996) identifies elementary school students' beliefs about the nature of math, themselves as math learners, the role of the math teacher, and how math is learned, as having such influences. Dweck (2000) discusses beliefs and theories about the self. Some studies address complex interactions among emotions, beliefs, achievement, and educational contexts. Wigfield & Eccles (2002) discuss the development of competence beliefs in relation to an expectancy-value model of achievement motivation. Pekrun et al. (2007, p. 25) note, "[C]ontrol-related beliefs (e.g., self-concepts of ability) and value-related beliefs (e.g., individual interests) can be assumed to affect appraisals and resulting achievement emotions… For example, if a student holds favorable control beliefs regarding her achievement in an academic domain like mathematics, an activation of these beliefs will lead to appraisals of challenging tasks as being manageable, and to related positive emotions." Here, we hypothesize that such beliefs also lead to a *lower threshold for activation* of particular engagement structures.

Articles in the book edited by Maass and Schlöglmann (2009) elaborate on the structured nature of beliefs generally, and beliefs pertaining to mathematics and its learning in particular. Here, Goldin et al. (2009) review ideas of beliefs as themselves structured (cf. Törner, 2002), as linking to each other to form belief systems (cf. Green, 1971), and as embedded in larger affective and cognitive structures.

Taking this work into account, we highlight the importance of describing the *particulars* of *how* beliefs, values, emotional feelings, and social situations interact in a structured way to influence in-the-moment engagement with mathematics. Thus, we point toward engagement structures as useful psychological constructs centered in the affective domain that embody some of these particulars explicitly (see Sect. 4).

(9) Interactions with the student's *self-identity*, *personality traits*, and *motivational orientations* are likewise hypothesized to influence the activation and continued in-the-moment influence of an engagement structure. Self-identity, as noted, also includes beliefs and theories about the self, as discussed by Dweck (2000) and others.

(10) Finally, we hypothesize characteristic interactions between the motivating desires, affective pathways, beliefs, and other strands of engagement structures, and the student's mathematical (as well as nonmathematical) *problem-solving strategies and heuristics*. Engagement structures can address *mathematical* engagement specifically—thus, the motivating desire for an intrinsic payoff in *Check This Out* may preferentially evoke exploratory problem-solving strategies; the motivating desire for task completion in *Get The Job Done* may evoke more procedural, time-efficient strategies or algorithms.

In their extensive discussion of motivation in mathematics education, Turner and Meyer review the literature on motivation generally, as well as that centered on mathematics. They comment,

> "In the educational psychology literature, the research is more focused on students and individual differences regarding their achievement goals, values, and efficacy for mathematics. In the mathematics education literature, the research questions are more likely to examine the processes through which students acquire mathematical understanding and problem solving skills. A third, smaller, literature has emerged from these two larger literatures to apply motivation theory to mathematics classrooms.
> "…We suggest that each of the three literatures could benefit from employing the strengths of the others, with a goal of studying motivation and mathematics not as separate or complementary, but as integrated, mutually constituted, and situated."
> (Turner & Meyer, 2009, p. 528)

Such an integrated study, which we see as highly desirable, must focus not only on students' longer-term motivations and orientations, but on the dynamics of their in-the-moment engagement.

Behind the issue of integrating motivation theory with mathematics education is the deeper psychological issue of how affect, cognition, and social interactions are understood to be related. Pioneering work in the psychology of emotion has focused on specific relations between affect and cognition (e.g., Csikszentmihalyi, 1990; Dai & Sternberg, 2004). Further, related discussions of affect with regard to mathematics learning may be found in Gomez-Chacon (2000); Hannula (2002); Lesh, Hamilton, & Kaput (2007); Zan, Brown, Evans, & Hannula (2006); Evans, Morgan, & Tsatsaroni (2006); and references therein. From such sources, we understand the affective domain to be both *individual* and *social*, to interact continuously with cognition, to function as a system that represents, encodes, and communicates information, to have immediate in-the-moment consequences for mathematical learning, and to have these effects in ways that essentially involve longer-term structures of beliefs, values, and orientations. Engagement structures characterize specific, hypothesized ways in which affect, cognition, and motivation interact to influence students' mathematical engagement in classroom social environments.

One might seek simply to consider such constellations as "frames," and see engagement structures as particular interpretive frames resulting from individuals' orientations interacting with their immediate, perceived cognitive and social environments. However, our terminology and perspective allow us to do more than examine framing—we are able to discuss many *specifics of the interacting components* that comprise engagement structures, from motivating desires to characteristic emotions, meta-affect, beliefs, social behavior, etc., much of which would not apply in other kinds of "framing" situations.

2.4 Examples of engagement structures

Next we describe nine specific examples constructed from observations, for which we have the best preliminary evidence. The list is not intended to be complete. For each structure, we highlight the motivating desire, the need (in Murray's sense) it may address, features of the social situation likely to evoke the desire, and some consequent behavior and/or emotions.

(1) "Get The Job Done": The student desires to complete an assigned mathematical task correctly following given instructions, thus fulfilling an implied obligation. The underlying need may be what Murray calls *deference*: "to yield to the influence of an allied other" [ideally in this case, the teacher] (p. 154). The desire is typically evoked by a teacher's class directions. Behavior is oriented toward straightforwardly carrying out the work. In group work, the student may enlist others in this goal. Emotional satisfaction accompanies fulfilling the obligation through task completion, not necessarily from mathematical learning. This structure is quite common in math classrooms.

(2) "Look How Smart I Am": The student desires to impress others (or, possibly, himself or herself) with his/her mathematical ability, knowledge, or genius. The need that may underlie the desire Murray terms *achievement*: "to increase self-regard by the exercise of talent" (p. 164). Evoking the desire may be a potentially admiring audience, or possibly the presence of "rivals" for high regard. In context, it may be expressive of a performance goal orientation. Behavior can be competitive, including "showing off" by trying to be fast or claiming one's own solution to be better than others. Emotional satisfaction accompanies achievement of recognition, if it occurs.

(3) "Check This Out": The motivating desire is to obtain a *reward* or "payoff"—immediately or in the future. *Intrinsic* rewards are associated with the "informing function" of reward, and *extrinsic* rewards with their "controlling function" (Zimmerman & Schunk, 2008). To function as a reward, something must have *value* to the student. One may distinguish a task's *intrinsic value* to a student (how interesting or enjoyable it is) from its *utility value* (how useful or potentially useful it is perceived as) (e.g., Wigfield & Eccles, 1992, 2000). The motivating desire for *Check This Out* may be based on intrinsic or utility value. The need behind the desire can vary with the nature of the reward. The desire is evoked situationally by perception of the payoff possibility. Behavior includes increased attention to the task in pursuit of the payoff. Fulfilling the desire may increase (intrinsic) interest in similar tasks or heighten (extrinsic) interest associating the math with the reward.

(4) "I'm Really Into This": Here the desire is to experience the very activity of addressing the task, ideally in "flow" (Csikszentmihalyi, 1990). The student is intrigued by the mathematics or the problem-solving process, "tuning out" other elements of the environment. Behind this desire (in the case of mathematics) may be the need Murray calls *understanding*: "to represent in symbols the order of nature" (p. 224). In context, it may express a mastery goal orientation. The opportunity presents itself with *social support* for deep engagement in a *challenging* problem. Satisfaction derives from achieving mathematical understanding, solving a difficult problem, or simply experiencing fascination.

(5) "Don't Disrespect Me": The motivating desire is to meet a perceived challenge or threat to the student's dignity, status, or sense of self-respect and well-being. The likely underlying need is termed by Murray's *infravoidance:* "to avoid conditions which may lead to belittlement" (p. 192). The social context may be a challenge perceived as belittling or insulting to the student's expression of a mathematical idea. Resistance to the challenge—defending oneself—raises the conflict to a new level. "Saving face" can override the issue of understanding the math, as the context becomes a highly charged discussion or argument.

(6) "Stay Out Of Trouble": The student desires to avoid interactions that may lead to conflict (e.g., a fight) or distress (e.g., embarrassment, humiliation, or anger) involving peers or someone in authority. Alternatively, the student may want to be left alone due to personal circumstances involving some emotional vulnerability. Murray describes the need for *harmavoidance:* "to take precautionary measures" (p. 197). The social context suggests to the student

possible trouble with others. Avoidance behavior, including striving not to be noticed, may supersede addressing the task's mathematical content. A sense of relief rewards success.

(7) "It's Not Fair": The motivating desire is to redress a perceived inequity. Underlying it may be the need Murray terms *succorance:* "to have one's needs gratified by an allied other" (p. 182). The desire is evoked as the student perceives some unfairness in a group problem-solving effort; e.g., with the level of participation by others, the role accorded to the student, or recognition from the teacher. Behavior ensues toward redressing the inequity, with likely disinvestment in the task itself. Satisfaction, if it occurs, can derive from restoring fairness, or else just "getting it done and over with."

(8) "Let Me Teach You": The student desires to help another understand or solve the problem. The need Murray identifies as *nurturance* includes: "to gratify the needs of a mentally confused person" (p. 184). The social situation evocative of this desire is one where the student becomes aware of someone who does not understand, while the student has an insight or relevant knowledge to share. Behavior includes trying to help by explaining or demonstrating, with satisfaction derived from the other student learning and/or appreciating the help.

(9) "Pseudo-Engagement": The motivating desire is to look good to the teacher or to peers by seeming to be engaged while avoiding genuine participation. The underlying need is termed by Murray *blame avoidance*: "to avoid blame or rejection" (p. 187). The desire arises when real mathematical participation is not perceived as possible or satisfying, but overt disengagement might evoke disapproval. Behavior includes trying to look busy or pretending to listen. Relief occurs as the activity ends without the student's detachment being noticed.

3 Further discussion

3.1 Desirability of engagement structures

In mathematics education, much of the study of affect—even that of emotions such as anxiety—focuses on trait-like aspects. *Attitudes*, understood either as propensities toward certain kinds of behavior or toward certain kinds of emotional feelings, are thought to change relatively slowly. Researchers usually study traits using survey or questionnaire instruments, such as the Mathematics Anxiety Rating Scale (Richardson & Suinn, 1972; see also Capraro et al., 2001), the Mathematics Attitudes Scales of Fennema and Sherman (1976), and similar measures. Typically, "positive" emotions and attitudes are taken as desirable, and "negative" ones as undesirable.

Likewise, *beliefs*, e.g., "control" and "value" beliefs, are seen as "trait-like student characteristics that in interaction with the classroom context are thought to influence [mathematical problem solving] processes" (Op 't Eynde et al., 2007, p. 191). Students are classifiable into "types", with Op 't Eynde et al.'s Mathematics-Related Beliefs Questionnaire, having "negative," "mildly positive," "positive," or "highly positive" belief profiles (p. 192). Students' beliefs are shown to be closely related to the emotions and perceptions they report during classroom mathematical problem solving, seen as a product of cognitive, affective, and conative processes.

We have already noted that constructive engagement does not exclusively involve "positive" emotional feelings, and that meta-affect can transform "negative" emotions so that they are experienced positively. Similarly, characterizing attitudes and beliefs as "positive" or "negative" can gloss over important ambiguities and complexities. Under some conditions, "negative" beliefs can contribute to constructive mathematical behaviors. The belief that math is difficult, and that one's own ability is limited, may most often foster disengagement; but in the right context—e.g., when high utility value is ascribed to success, and/or when a respected role model provides inspiration—such beliefs can evoke determination, commitment to the hard work needed to overcome one's limitations, and great persistence. We must be able to study how such contexts occur.

In contrast to the classification of emotional feelings, attitudes, and beliefs as positive or negative, we stress that *we do not regard some engagement structures as "good" and others as "bad."* We see them as universally or near-universally present in individuals. Each regulates affect, cognition, and social behavior in a way that can be adaptive. Even *Pseudo-Engagement* can function constructively in a classroom, allowing the painfully bored or alienated student to engage non-disruptively with something other than the task at hand. The challenge for the math teacher is to create an *emotionally safe* environment, with serious engagement based on many different, but appropriately active, structures that contribute to interest, utility, safety, status, self-image, self-concept, and understanding; ultimately, to fulfilling basic psychological needs.

3.2 Stages in active engagement structures

We propose for each structure an idealized description of the stages that occur as it plays itself out successfully in a classroom context, focusing on emotional feelings, self-talk, and strategies for fulfilling the motivating desire. These are: (A) *initial activation* of the structure; (B) *initial behaviors* toward achieving the motivating desire, and strategy for fulfilling it; (C) the *continuation*, including (successful) implementation of the strategy; and (D) the *outcome*, (ideally) achievement of the object of the motivating desire, and consequences for mathematics learning.

3.3 Specificity and universality of engagement structures

While some of the motivating desires in specific engagement structures are consonant with particular achievement goals, most of them are more closely connected with the social aspects of learning in the classroom context and their evocation is highly context dependent. Consequently, they are not easily classified within the theoretical frameworks of the educational psychology literature.

Furthermore, engagement structures per se are not specific to just one social context. Our focus on mathematical engagement in classrooms has led us to describe structures aligned with a particular set of motivating desires. These can be activated in other contexts too. Also, many different motivating desires arise in the everyday lives of middle school children, so that engagement structures not discussed here most probably come into play.

The structures we propose are not specific to any particular cultural, racial, ethnic, or economic groups. Some of the face-saving issues central to *Don't Disrespect Me* are described in studies of inner-city street life, but the same engagement structure can be inferred from behavior in college faculty meetings, situations of difficult political negotiations, formal social gatherings, etc. The particular expressions of affect—the fourth strand in Sect. 2.3—can differ substantially according to sociocultural norms and across individuals; but we anticipate the underlying *structure* to be essentially invariant. In this sense, we regard engagement structures as *archetypal*, and *not stereotypical*.

3.4 Branch points in engagement structures

Let us take further the analogy between engagement structures and cognitive structures. During problem solving, a student employs heuristic processes and strategies. As this plays out, preestablished cognitive structures or schemata become active. For example, a governing strategy of systematic trial and error, in a situation where two constraints are imposed in a mathematical problem, may entail "activating" simultaneous coordination of conditions. There may then occur an opportunity to draw on proportional reasoning, e.g., if one of the conditions being coordinated is multiplicative. As the problem solving proceeds, the solver might notice a more direct solution method using proportionality, without further trial and error, and abandon the coordination of conditions. One might describe this process as *branching* from one active cognitive structure to a different one. A structure initially accessed as a kind of "subroutine" becomes the governing one.

Similarly, we notice what seem to be critical "branch points" in engagement structures. These occur when someone can (consciously or otherwise) *change the motivating desire*, thus activating a different structure and experiencing different thoughts and feelings. Particularly when events unfold in an unexpected or undesired way (e.g., due to classmates' responses or the teacher's interventions), the original structure may become inactive and another may replace it.

For instance, a peer's challenge to a student's work can elicit *Don't Disrespect Me*, with the student becoming defensive of her position to the point of unwillingness to actually consider the reasoning of the other student. Subsequent comments are construed as "attacks" on her mathematical or social identity. As the student defends her ideas, however, she may come to feel sufficiently secure that she begins to take seriously the arguments of the student who challenged her. If something in those comments suggests a possible payoff—e.g., by offering a new perspective to the problem—her subsequent responses may be more consistent with *Check This Out*, with *Don't Disrespect Me* no longer active.

Likewise, a student may set out to *Get The Job Done*, but notice along the way that he understands something another student does not. Initially, *Let Me Teach You* becomes active in service of his group's task completion. As the student becomes more engaged with teaching his peer, the imparting of understanding may become the major motivating desire, with the goal of simply completing the assigned task no longer salient: *Get The Job Done* has branched into *Let Me Teach You*. Alternatively, as one student attempts to teach another, his peer may not regard him as especially knowledgeable or smart and reject the help. *Let Me Teach You* may branch into *Look How Smart I Am*, as the first student tries to impress his peer with his knowledge or ability. The latter, accessed initially in service of *Let Me Teach You*, may come to govern the engagement.

In short, engagement structures within each individual not only call upon each other, but can supersede each other in the course of the changing social situation.

3.5 "Fit" with other theoretical ideas

Ross and Nisbett (1991) consider modern social psychology to rest on a "tripod" of concepts derived from ideas that go back to Kurt Lewin, termed "the father of American social psychology" (Jost & Kay, 2010, p. 1123). This tripod consists of: (1) the principle of *situationism*, (2) the principle of *construal*, and (3) the concept of *tension systems*. These three concepts fit well with the way we think about engagement structures. Their dynamical nature and their activation in students in specific social situations are consistent with situationism—the student's state and behavior changing from moment to moment and dependent on how the situation is construed. This notion of construal underlies the concept of schema or knowledge structure that "summarizes generic knowledge and previous experience with respect to a given class of stimuli and events and, at the same time, gives meaning and guides anticipation with respect to similar stimuli and events in the future" (Ross & Nisbett, 1991 p. 12). The power of a construed situation sets up a tension system that calls for resolution. Such tensions can "swamp" individuals' motivational orientations, evoking a wide spectrum of "in the moment" motivating desires and consequent engagement structures.

For example, imagine two (idealized) students with different academic goal orientations—the first mastery oriented and the second performance oriented, with belief structures supporting those orientations. As class begins, Irene is *more likely* to find a math problem intrinsically interesting, and to want to understand the underlying concepts, while Michael is *more likely* to want to solve the problem to show classmates or the teacher that he is smart. The engagement that ensues is guided by Irene's structures of *Check This Out* and *I'm Really Into This*, and Michael's structure of *Look How Smart I Am*. A little later, after they have presented their thinking, Richard (a third student) calls out, "Neither of you knows what you're talking about!" The situation now is profoundly different. The power of the new situation can "swamp" that of the goal orientations, creating a tension system that begs for resolution. Richard's challenge changes the "life space" (in Lewin's sense) for Michael and Irene. How it does so depends on how each construes Richard's statement and nonverbal expressions. Insofar as the comment is simply a string of eight words, it is an "objective stimulus;" but for Michael and Irene, it carries unique personal meanings. There may follow responses guided by *Don't Disrespect Me* or *Stay Out of Trouble*; *Check This Out* may branch into *Look How Smart I Am*, and there are other possibilities as well.

Thus, while "trait" variables importantly influence behavior, our study of engagement structures leads us to share the perspective of most social psychologists that the immediately construed situation has still greater influence.

4 Beliefs intertwined with and acting through engagement structures

Now, we consider the relation of individuals' beliefs and belief structures, and their associated values, to engagement structures. Recall that the latter are taken as present in *most or all* individuals; their activation is descriptive of the person's *state*. Beliefs are taken as *specific* to individuals, propositions held as true or valid, and (except for highly transient beliefs) as *traits*.

Beliefs may be warranted through reasoning, experience, or evaluation of evidence, and are normally supported by emotional feelings and meta-affect. They may meet emotional needs, providing (for example) defense from pain. We have noted research that supports the relationship of "control" and "value" beliefs to motivational orientations, as well as to emotions experienced during problem solving. We hypothesize that:

1. By influencing how the student construes the situation, beliefs (and, more generally, orientations) strongly affect the selection of a motivating desire for arousal and consequently an engagement structure for activation.
2. Subsequently, beliefs influence the student's in-the-moment interactions, as s/he behaves in characteristic ways s/he judges move toward satisfying the motivating desire.
3. Finally, the outcome of activating the engagement structure can feed back to confirm (or, more rarely, call into question) related beliefs.

Thus, engagement structures suggest mechanisms whereby beliefs have reciprocal influences on in-the-moment mathematical engagement.

The first hypothesis reflects how trait-like characteristics help direct attention preferentially toward features of the environment taken to be salient. Thus, two individuals with different beliefs in similar environments have different potentials for activating particular engagement structures. For example, a female student (idealized) holds the *belief* that her teacher thinks "girls are not as good at math as boys." When the teacher calls on a boy instead of her, she *attends* to this and *attributes* it to what she believes is the teacher's bias. This increases the likelihood of activating the It's Not Fair structure—particularly, if the girl thinks she has a good answer. She engages subsequently in actions toward fulfilling her *motivating desire* to restore fairness. To a classmate without this belief, the teacher's action is not salient but, for her, it channels her energy into fulfilling a social goal.

The second hypothesis points to how beliefs can affect the evaluation of what action(s) can or should be taken toward satisfying a desire. Thus, the intertwined beliefs affect the nature of the subsequent engagement with the math. In the example described, the girl's *belief* that she has high mathematical ability can suggest a strategy of "proving herself" to the teacher to rectify the perceived injustice. The engagement structure *Look How Smart I Am* is activated in service of the *It's Not Fair* structure. She tries to excel by solving the next problem in a way her teacher cannot ignore. Without this self-efficacy belief, she might find a different strategy for restoring fairness or possibly abandon her motivating desire.

The third hypothesis suggests that interactions between beliefs and the other intertwined strands of engagement structures are bidirectional. If the strategy of "proving herself" has the desired result, the student's self-efficacy beliefs may strengthen. If the teacher actually acknowledges her ability in an extraordinary way, a new meta-affective context may be established, and even the girl's original belief about the teacher's bias may change. In the absence of such outcomes, she may disinvest in further conceptual learning, branching into *Get The Job Done* or even *Pseudo-Engagement*. Her original belief in the teacher's bias is confirmed, but a very negative outcome may call her self-efficacy beliefs into question.

Similar possibilities exist, e.g., for an African-American student who believes that teachers favor white or Hispanic students.

A common *belief system* involves a student's belief that mathematical ability is inborn and innate. Failure or mediocre performance is then "not my fault." The student takes some pride in saying, "I am just not a math person—I wasn't born with it." The belief is a part of the student's mathematical identity. Related beliefs may include the ideas that: mathematical success is just a matter of knowing the right procedures; high ability means being able to remember rules easily and use them rapidly and accurately; test scores reflect ability so that s/he cannot (therefore) normally expect a high grade. This belief structure may function to assuage guilt, providing the student under some conditions with good reasons to disengage before frustration can arise.

Under common classroom conditions, such beliefs may support arousal of the motivating desire or actions toward fulfilling the desire in engagement structures such as *Get The Job Done, Stay Out Of Trouble*, or *Pseudo-Engagement*, while correspondingly inhibiting activation of structures such as *I'm Really Into This, Look How Smart I Am*, or *Let Me Teach You*.

For instance, *Get The Job Done* allows a student with such beliefs to use his knowledge to complete the task procedurally, if possible; to ask for step-by-step help from the teacher or a peer, and to detach from further cognitive engagement, all *without calling his beliefs into question*. In contrast, *I'm Really Into This*—should it occur—might threaten these beliefs, forcing an emotionally unwelcome change in his mathematical identity. Thus, the belief structure facilitates one engagement structure and impedes another when the student faces a conceptually challenging math problem in class. This does *not* mean that he lacks the *I'm Really Into This* structure. It may well become active while playing sports or acting in a school play, but it is just less likely in the context of mathematics.

Such examples suggest identifying, for each engagement structure, commonly held mathematical beliefs or belief systems and values that can facilitate activating the structure in some situations. Here are some likely candidates:

Get The Job Done: Math is mainly procedural, answer oriented, and rule governed, requiring thoroughness, with the teacher giving directions as the authority. Compliance and meeting expectations are valued.

Look How Smart I Am: Math requires high innate ability or genius, and others think so too. The student holds high self-efficacy beliefs and values mathematical ability.

Check This Out: Math has internal logic, inherent interest, some valuable areas of application, and/or is useful to achieve other goals. The student believes s/he can achieve a perceived reward by working on the problem, valuing the reward, and possibly doing conscientious work.

I'm Really Into This: Math, mathematical representation, and/or problem solving are intriguing, with internal logic and coherence. The student's self-concept is as an effective problem solver, serious, an engaged thinker. Problem-solving or learning activity is valued for its own sake.

Don't Disrespect Me: Correctness of answers or reasoning is important to status, which is highly valued. The student's self-concept includes capability of assertiveness and entitlement to respect.

Stay Out Of Trouble: Math or class activity can be dangerous or strewn with pitfalls. The student's self-concept includes low capability of self-defense if challenged, or low emotional or intellectual self-efficacy beliefs. Conflict avoidance is highly valued.

It's Not Fair: School activity entails implicit rules of fairness in division of work and bestowal of acknowledgment. Bias exists in recognizing individuals' or groups' abilities, contributions, and rights. Equality of treatment and sharing fairly are highly valued.

Let Me Teach You: Math has understandable internal logic, and the student has high self-efficacy beliefs. Understanding and helping others are both valued.

Pseudo-Engagement: Math is difficult and/or inaccessible, unpleasant, boring, or too easy. The teacher attends to or values mainly outward signs of engagement and compliance. The student has low self-efficacy beliefs, or possibly a high but unwarranted self-concept. Satisfactory opinions of others, or avoidance of low opinions, are valued.

5 Further research and potential implications

The ideas set forth here, while theoretically motivated and empirically suggested, are still at a relatively early stage of development.

We have suggested that engagement structures meet or can meet applicable scientific criteria. They address the *need* for a "mid-level" construct, centered in the affective domain, which describe rich details of students' in-the-moment mathematical behavior and permit characterization of possible mechanisms through which beliefs influence classroom engagement. In these respects, they offer *theoretical utility*. We have suggested a *fit* with existing theoretical constructs in the theory of affect, goals, motivation, and beliefs. Most centrally, engagement structures help us understand how achievement orientations, values, and beliefs can influence the immediate, highly variable nature of students' behavioral, cognitive, and affective engagement. We have situated our work in perspectives from social psychology that emphasize the overriding importance of the situation as construed by the individual.

Of course, we note important limitations. While the hypothetical components of engagement structures are theoretically motivated, the specific structures proposed emerge from qualitative observations in a limited number of classrooms. Their descriptions are still fluid and subject to modification.

At this stage of research, important empirical questions are suggested by the engagement structure construct and its connection with beliefs: Can particular structures be empirically validated, and can their activation be reliably observed and measured? What specific roles do they play in developing self-efficacy and other beliefs pertaining to mathematics and in influencing motivational orientations? An obvious direction of inquiry is the relation between structures inferred as active during mathematics classes and students' prior and subsequent beliefs and orientations. What teacher interventions affect development and activation of particular engagement structures in mathematical contexts, and how? Is there differential activation of structures in schools with different cultural and socioeconomic characteristics, or in response to different kinds of mathematical tasks? How do mathematics teachers respond to discussions of engagement at this level in professional development contexts, and how do their responses depend on their beliefs? We thus see the construct as susceptible to validation, suggestive of future directions, and having *empirical utility*.

We also see great potential *practical value*. We suggest that important results will ensue from the finer-grained study of students' classroom mathematical behavior with these tools. As we identify the most important engagement structures and characterize them more precisely, we shall learn what to take as persuasive evidence that a particular structure is functioning. We can then learn how to recognize and influence the choices students make at the most critical branch points. This leads toward mathematics teaching strategies that foster optimal kinds of situation-specific engagement. Developing the language of engagement structures and their relation to beliefs facilitates professional discussions with mathematics teachers, promoting explicit awareness as teachers recognize many of the social/behavioral patterns described. Our work to this point in inner-city classrooms suggests the value of such a program.

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References

Abelson, R. P. (1981). Psychological status of the script concept. *American Psychologist, 36*(7), 715–729.

Alston, A., Goldin, G. A., Jones, J., McCulloch, A., Rossman, C., & Schmeelk, S. (2007). The complexity of affect in an urban mathematics classroom. In T. Lamberg & L. R. Wiest (Eds.), *Exploring mathematics education in context: Proceedings of the 29th annual meeting of PME-NA, Lake Tahoe, NV* (pp. 326–333). Reno: University of Nevada.

Anderman, E. M., & Wolters, C. A. (2006). Goals, values, and affect: Influences on student motivation. In P. Alexander & P. Winne (Eds.), *Handbook of educational psychology* (pp. 369–389). Mahwah, NJ: Erlbaum.

Anderson, E. (1999). *Code of the street*. NY: W. W. Norton.

Beck, A. T. (1976). *Cognitive therapy and the emotional disorders*. NY: New American Library.

Capraro, M. M., Capraro, R. M., & Henson, R. K. (2001). Measurement error of scores on the mathematics anxiety rating scale across studies. *Educational and Psychological Measurement, 61*, 373–386.

Cattell, R. B., & Scheier, I. H. (1961). *The meaning and measurement of neuroticism and anxiety*. NY: Ronald Press.

Csikszentmihalyi, M. (1990). *Flow: The psychology of optimal experience*. NY: Harper & Row.

Dai, D. Y., & Sternberg, R. J. (Eds.). (2004). *Motivation, emotion, and cognition: Integrative perspectives on intellectual functioning and development*. Mahwah, NJ: Erlbaum.

Dance, L. J. (2002). *Tough fronts: The impact of street culture on schooling*. NY: Routledge-Falmer.

Davis, R. B. (1984). *Learning mathematics: The cognitive science approach to mathematics education*. Norwood, NJ: Ablex.

DeBellis, V. A., & Goldin, G. A. (2006). Affect and meta-affect in mathematical problem solving: A representational perspective. *Educational Studies in Mathematics, 63*(2), 131–147.

Dweck, C. S. (2000). *Self-theories: Their role in motivation, personality, and development*. Philadelphia, PA: Taylor & Francis.

Epstein, Y. M., Goldin, G. A., Schorr, R. Y., Capraro, R., Capraro, M. M., & Warner, L. B. (2010). *Measuring engagement structures in middle-grades urban mathematics classrooms*. Paper presented at the 2010 annual meeting of AERA, Denver, CO.

Epstein, Y., Schorr, R. Y., Goldin, G. A., Warner, L., Arias, C., Sanchez, L., et al. (2007). Studying the affective/social dimension of an inner-city mathematics class. In T. Lamberg & L. R. Wiest (Eds.), *Exploring mathematics education in context: Proceedings of the 29th annual meeting of PME-NA, Lake Tahoe, NV* (pp. 649–656). Reno: University of Nevada.

Evans, J., Morgan, C., & Tsatsaroni, A. (2006). Discursive positioning and emotion in school mathematics practices. *Educational Studies in Mathematics, 63*(2), 209–226.

Fennema, E., & Sherman, J. A. (1976). Fennema-Sherman Mathematics Attitudes Scales: Instruments designed to measure attitudes toward the learning of mathematics by females and males. *Journal for Research in Mathematics Education, 7*, 324–326.

Finn, J. D. (1993). *School engagement and students at risk*. Washington, DC: U. S. Department of Education, National Center for Educational Statistics (ERIC Document Reproduction Service No. ED 362 322).

Flavell, J. H. (1979). Metacognition and cognitive monitoring: A new area of cognitive-developmental inquiry. *American Psychologist, 34*(10), 906–911.

Fredricks, J. A., Blumenfeld, P. C., & Paris, A. H. (2004). School engagement: Potential of the concept, state of the evidence. *Review of Educational Research, 74*(1), 59–109.

Furinghetti, F., & Pehkonen, E. (2002). Rethinking characterizations of beliefs. In G. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 39–57). Dordrecht: Kluwer.

Goffman, E. (1974). *Frame analysis*. NY: Harper Colophon.

Goldin, G. A. (2000). Affective pathways and representation in mathematical problem solving. *Mathematics Thinking and Learning, 2*, 209–219.

Goldin, G. A. (2002). Affect, meta-affect, and mathematical belief structures. In G. Leder, E. Pehkonen, & G. Tőrner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 59–72). Dordrecht: Kluwer.

Goldin, G. A., Epstein, Y. M., & Schorr, R. Y. (2007). Affective pathways and structures in urban students' mathematics learning. In D. K. Pugalee, A. Rogerson, & A. Schinck (Eds.), *Mathematics education in a global community: Proceedings of the 9th international conference of the mathematics education into the 21st century project* (pp. 260–265). Charlotte, NC: University of North Carolina.

Goldin, G. A., Rösken, B., & Törner, G. (2009). Beliefs: No longer a hidden variable in mathematics teaching and learning processes. In J. Maass & W. Schlöglmann (Eds.), *Beliefs and attitudes in mathematics education: new research results* (pp. 1–18). Rotterdam: Sense.

Gomez-Chacon, I. M. (2000). Affective influences in the knowledge of mathematics. *Educational Studies in Mathematics, 43*, 149–168.

Green, T. F. (1971). *The activities of teaching*. Tokyo: McGraw-Hill.

Greeno, J. G., & Goldman, S. V. (1988). *Thinking practices in mathematics and science learning*. Mahwah, NJ: Erlbaum.

Greenwood, C. R. (1991). Longitudinal analysis of time, engagement, and achievement in at-risk versus non at-risk students. *Exceptional Children, 57*, 521–535.

Grouws, D. A., & Lembke, L. O. (1996). Influential factors in student motivation to learn mathematics: The teacher and classroom culture. In M. Carr (Ed.), *Motivation in mathematics* (pp. 39–62). Cresskill, NJ: Hampton Press.

Hannula, M. S. (2002). Attitude towards mathematics: Emotions, expectations and values. *Educational Studies in Mathematics, 49*, 25–46.

Jost, J. T., & Kay, A. C. (2010). Social justice: History, theory, and research. In S. T. Fiske, D. T. Gilbert, & G. Lindzey (Eds.), *Handbook of social psychology* (5th ed., Vol. 2, pp. 1122–1165). Hoboken, NJ: Wiley.

Kloosterman, P. (1996). Students' beliefs about knowing and learning mathematics: Implications for motivation. In M. Carr (Ed.), *Motivation in mathematics* (pp. 131–156). Cresskill, NJ: Hampton Press.

Laurent, J., Catanzaro, S. J., Joiner, T. E., Jr., Rudolph, K. D., Potter, K. I., Lambert, S., et al. (1999). A measure of positive and negative affect for children: Scale development and preliminary validation. *Psychological Assessment, 11*(3), 326–338.

Leder, G. C., Pehkonen, E., & Törner, G. (Eds.). (2002). *Beliefs: A hidden variable in mathematics education?* Dordrecht: Kluwer.

Lesh, R., Hamilton, E., & Kaput, J. J. (Eds.). (2007). *Foundations for the future in mathematics education*. Mahwah, NJ: Erlbaum.

Linnenbrink, E. A. (2007). The role of affect in student learning: A multi-dimensional approach to considering the interaction of affect, motivation, and engagement. In P. A. Schutz & R. Pekrun (Eds.), *Emotion in education* (pp. 107–124). London: Elsevier.

Linnenbrink, E. A., & Pintrich, P. R. (2000). Multiple pathways to learning and achievement: The role of goal orientation in fostering adaptive motivation, affect, and cognition. In C. Sansone & J. M. Harackiewicz (Eds.), *Intrinsic and extrinsic motivation: The search for optimal motivation and performance* (pp. 195–227). San Diego, CA: Academic Press; cited in Anderman & Wolters (2006).

Linnenbrink, E. A. & Pintrich, P. R (2002). Achievement goal theory and affect: An asymmetrical bidirectional model. *Educational Psychologist, 37*, 69–78; cited in Anderman & Wolters (2006).

Maass, J., & Schlöglmann, W. (Eds.). (2009). *Beliefs and attitudes in mathematics education: New research results*. Rotterdam: Sense.

Malmivuori, M.-L. (2006). Affect and self-regulation. *Educational Studies in Mathematics, 63*, 149–164.

Marks, H. M. (2000). Student engagement in instructional activity: Patterns in the elementary, middle, and high school years. *American Educational Research Journal, 37*, 153–184.

McLeod, D. B. (1994). Research on affect and mathematics learning. *Journal for Research in Mathematics Education, 25*, 637–647.

McLeod, D. B., & McLeod, S. H. (2002). Synthesis—beliefs and mathematics education: Implications for learning, teaching, and research. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 115–123). Dordrecht: Kluwer.

Middleton, J. A., & Spanias, P. A. (1999). Motivation for achievement in mathematics: Findings, generalizations, and criticisms of the research. *Journal for Research in Mathematics Education, 30*(1), 65–88.

Murray, H. A. (2008). *70th Anniversary edition, explorations in personality*. NY: Oxford University Press.

Op 't Eynde, P., De Corte, E., & Verschaffel, L. (2002). Framing students' mathematics-related beliefs. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 13–37). Kluwer: Dordrecht.

Op 't Eynde, P., De Corte, E., & Verschaffel, L. (2007). Students' emotions: A key component of self-regulated learning? In P. Schutz & R. Pekrun (Eds.), *Emotion in education* (pp. 185–204). Burlington, MA: Academic Press.

Pajares, M. F. (1992). Teachers' beliefs and educational research: Cleaning up a messy construct. *Review of Educational Research, 62*, 307–332.

Pajares, M. F., & Graham, L. (1999). Self-efficacy, motivation constructs, and mathematics performance of entering middle school students. *Contemporary Educational Psychology, 24*, 124–139.

Park, S. (2005). Student engagement and classroom variables in improving mathematics achievement. *Asia Pacific Education Review, 6*(1), 87–97. doi:10.1007/BF03024970.

Pekrun, R., Frenzel, A. C., Goetz, T., & Perry, R. P. (2007). The control-value theory of achievement emotions: An integrative approach to emotions in education. In P. Schutz & R. Pekrun (Eds.), *Emotion in education* (pp. 13–36). Burlington, MA: Academic Press.

Pintrich, P. R. (2000). Multiple goals, multiple pathways: The role of goal orientation in learning and achievement. *Journal of Educational Psychology, 92*, 544–555.

Richardson, F., & Suinn, R. (1972). The mathematics anxiety rating scale: Psychometric data. *Journal of Counseling Psychology, 19*, 551–554.

Rogers, T. B. (1983). Emotion, imagery, and verbal codes: A closer look at an increasingly complex interaction. In J. Yuille (Ed.), *Imagery, memory, and cognition: Essays in honor of Allan Paivio* (pp. 295–305). Mahwah, NJ: Erlbaum.

Rokeach, M. (1968). *Beliefs, attitudes, and values: A theory of organization and change*. San Francisco: Jossey-Bass.

Rokeach, M. (1973). *The nature of human values*. NY: The Free Press.

Ross, L., & Nisbett, R. E. (1991). *The person and the situation: Perspectives of social psychology*. NY: McGraw-Hill.

Schoenfeld, A. H. (1985). *Mathematical problem solving*. NY: Academic Press.

Schoenfeld, A. H. (2010). *How we think: A theory of goal-oriented decision making and its educational applications*. NY: Routledge.

Schorr, R. Y., Epstein, Y. M., Warner, L. B., & Arias, C. C. (2010a). Don't disrespect me: Affect in an urban math class. In R. Lesh, P. L. Galbraith, C. R. Haines, & A. Hurford (Eds.), *Modeling students' mathematical modeling competencies: ICTMA 13* (pp. 313–325). NY: Springer.

Schorr, R. Y., Epstein, Y. M., Warner, L. B., & Arias, C. C. (2010b). Mathematical truth and social consequences: The intersection of affect and cognition in a middle school classroom. *Mediterranean Journal for Research in Mathematics Education, 9*, 107–134.

Schorr, R. Y., & Goldin, G. A. (2008). Students' expression of affect in an inner-city SimCalc classroom. *Educational Studies in Mathematics, 68*, 131–148.

Schunk, D. H. (2001). Social-cognitive theory and self-regulated learning. In B. Zimmerman & D. Schunk (Eds.), *Self-regulated learning and academic achievement: Theoretical perspectives* (2nd ed., pp. 125–151). Mahwah, NJ: Erlbaum; cited in Anderman & Wolters (2006).

Schunk, D. H., & Zimmerman, B. J. (Eds.). (2008). *Motivation and self-regulated learning: Theory, research, and applications*. NY: Routledge.

Schutz, P., & Pekrun, R. (Eds.). (2007). *Emotion in education.* Burlington, MA: Academic Press (Elsevier).

Skinner, E., Kindermann, T., & Furrer, C. (2009). A motivational perspective on engagement and disaffection: Conceptualization and assessment of children's behavioral and emotional participation in academic activities in the classroom. *Educational and Psychological Measurement, 69*, 493–525.

Törner, G. (2002). Mathematical beliefs—a search for a common ground. In G. C. Leder, E. Pehkonen, & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 73–94). Dordrecht: Kluwer.

Turner, J. C., & Meyer, D. K. (2009). Understanding motivation in mathematics: What is happening in classrooms? In K. R. Wentzel & A. Wigfield (Eds.), *Handbook of motivation at school* (pp. 527–552). NY: Routledge.

van de Sande, C., & Greeno, J. G. (2010). A framing of instructional explanations: Let us explain *with* you. In M. K. Stein & L. Kucan (Eds.), *Instructional explanations in the disciplines* (pp. 69–82). NY: Springer.

Wellborn, J. G. (1991). *Engaged and disaffected action: The conceptualization and measurement of motivation in the academic domain*. Unpublished doctoral dissertation, University of Rochester, NY; cited in Skinner et al. (2009).

Wigfield, A., & Eccles, J. S. (1992). The development of achievement task values: A theoretical analysis. *Developmental Review, 12*, 265–310.

Wigfield, A., & Eccles, J. S. (2000). Expectancy-value theory of achievement motivation. *Contemporary Educational Psychology, 25*, 68–81.

Wigfield, A., & Eccles, J. S. (2002). The development of competence beliefs, expectancies for success, and achievement values from childhood through adolescence. In A. Wigfield & J. S. Eccles (Eds.), *Development of achievement motivation* (pp. 91–120). San Diego, CA: Academic Press.

Zajonc, R. B. (1980). Feeling and thinking: Preferences need no inferences. *American Psychologist, 35*, 151–175.

Zan, R., Brown, L., Evans, J., & Hannula, M. S. (2006). Affect in mathematics education: An introduction. *Educational Studies in Mathematics, 63*(2), 113–121.

Zimmerman, B. J., & Schunk, D. H. (2008). Motivation: An essential dimension of self-regulated learning. In D. H. Schunk & B. J. Zimmerman (Eds.), *Motivation and self-regulated learning: Theory, research, and applications* (pp. 1–30). NY: Routledge.